Optimal Monetary Policy in a New Keynesian Model with Endogenous Growth*

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Abstract

We study optimal monetary policy in a New Keynesian (NK) model with endogenous growth and knowledge spillovers external to each firm. We find that, in contrast with the standard NK model, the Ramsey dynamics implies deviation from full inflation targeting in response to technology and government spending shocks, while the optimal operational rule is backward looking and responds to inflation and output deviations from their long-run levels.

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1 Introduction

The traditional NK literature features exogenous growth or no growth at all. Yet, business cycle fluctuations affect growth-enhancing activities and modify the growth trend of the entire economy. However, very few papers analyze the interaction between growth and uncertainty in the context of monetary models (e.g. Dotsey and Sarte 2000 and Varvarigos 2008). An even smaller subset introduce nominal rigidities, but in the form of one-period nominal wage contracts.[1] An exception are Annicchiarico et al. (2011b), who consider an NK model and study the interplay between nominal rigidities, nominal uncertainty and growth under different Taylor rules, but do not study optimal monetary policy. Those papers studying optimal monetary policy in NK frameworks, instead, (e.g. Khan et al. 2003, Schmitt-Grohé and Uribe 2004a, 2007a, 2007b, Faia 2008a, 2008b, 2009), usually abstract from growth, so disregarding the interaction between short-run dynamics and growth which is, instead, of interest for the optimal monetary policy analysis.[2] The paper most related to ours is Faia (2008b), which studies Ramsey monetary policy in a basic NK model with capital accumulation and sticky prices *à la* Rotemberg (1982), but no growth.

In this paper we fill this gap and consider an NK model with endogenous growth *à la* Romer (1986) and nominal rigidities due to staggered prices *à la* Calvo (1983) to study optimal monetary policy. Since in this paper we want to deviate from the mainstream NK model only for the inclusion of an endogenous growth mechanism, we opt for the Calvo setting which among the various models of price rigidities is the most widely used in the derivation of New Keynesian Phillips Curves and represents a key ingredient of the standard NK textbook model (see e.g. Galí 2008 and Walsh 2010). Moreover, Ascari et al. (2011) provide evidence in favor of the statistical superiority of the Calvo setting with respect to the Rotemberg one.[3]

In particular, we study the Ramsey optimal monetary policy and characterize the monetary policy rules that are optimal within a family of implementable and simple rules in a calibrated model of the business cycle under a positive steady-state inflation rate. In this respect we depart from the standard NK literature which studies optimal monetary policy in economies where long-run inflation is nil or there is some form of wide-spread

[1] Blackburn and Pelloni (2004, 2005) and Annicchiarico et al. (2011a).

[2] An exception is given by Mattesini and Nisticò (2010) who explore the optimal behavior of the monetary authorities in an NK model with trend (exogenous) growth.

[3] We are aware that the Calvo and Rotemberg price-setting mechanisms, despite the strong similarities to a first order of approximation (provided that there is no trend inflation, as shown by Ascari and Rossi 2012), may have very different welfare implications at higher order of approximation, even if almost negligible,when the steady state is distorted (see e.g. Lombardo and Vestin 2008). Hence, for robustness check and to make our findings more comparable to those of Faia (2008b), in a separate appendix, available on the authors' webpages, we also consider an NK model with AK technology and Rotemberg pricing.

indexation.[4] From an empirical point of view, neither of these two assumptions is realistic for economies like the United States or the Euro Area. Thus, it is of interest to investigate the characteristics of optimal policy in their absence and their relationship with growth.

The economy we consider in this paper features three sources of inefficiency which provide a rationale for the conduct of monetary policy. The first two distortions are the ones which characterize the basic NK model, namely: (i) monopolistic competition, which generates an average markup, which lowers output with respect to the efficient economy; (ii) nominal rigidities due to staggered prices, which generate price dispersion. The third source of inefficiency is the one that differentiates the present model from the standard NK model, i.e. the presence of *knowledge spillovers* which are external to each firm. In other words, a sort of serendipitous learning mechanism characterizes the production activity. In this context, the decentralized equilibrium is Pareto suboptimal and the economy grows at a lower rate than under the allocation that would maximize the representative household's lifetime utility. The following main results characterize our model economy.

First, even in the presence of the additional distortion due to knowledge spillovers, we find that the Ramsey steady-state inflation rate is zero.[5] The reason is the following. In the long run, a higher inflation rate, by increasing the average markup and by introducing price dispersion, would imply a lower return on capital and a reduced level of economic activity, thus lowering savings and growth. The increase in consumption and in growth rate more than compensates the increase in hours worked and thus households' welfare increases as trend inflation decreases.

Second, despite the long-run value of inflation is zero, the Ramsey dynamics requires deviation from full inflation targeting in response to technology and government spending shocks. However, the intensity of the reaction crucially depends on the nature of the shock. Following a positive technology shock the central bank tolerates moderate deviations of the inflation rate from its optimal steady state in order to push the short-run economy growth rate toward the efficient one. In this case optimality calls for an increase in the real interest rate so as to moderate consumption, foster capital accumulation and so growth.[6] Also in response to a government spending shock, the optimal monetary

[4] An exception are Schmitt-Grohé and Uribe (2007a). The authors show that, by assuming zero steady-state inflation or full price indexation, nominal rigidities have no real consequences for economic activity and thus welfare in the long run. Thus, the assumptions of zero long-run inflation or indexation should not be expected to be inconsequential for the form that optimal monetary policy takes.

[5] It is well known in the literature that in a model with Calvo pricing the first two distortions require a zero steady-state inflation (see King and Wolman 1999).

[6] We will see that this result stands in sharp contrast with that obtained by Faia (200b) in the context of a NK model with capital accumulation. In this sense we argue that the explicit introduction of an endogenous growth mechanism itself may be source of non-trivial implications for the optimal monetary policy analysis. As already anticipated, for robustness check, we have also solved the Ramsey problem under Rotemberg pricing and found that the optimal steady-state inflation is zero (as in Faia 2008b), while the optimal response to productivity and public consumption shock is always counter-cyclical.

requires an increase in the real rate, so as to generate a fall in consumption and mitigate the expansionary effects of the demand shock.

Finally, the optimal operational monetary rule is backward-looking, features a strong positive reaction to output movements and a mild response to inflation, contrary to the previous findings in the literature.[7] As will be clear in the paper, all these results strongly depend on the role played by the endogenous growth mechanism and the implied inefficiency due to the presence of external knowledge spillovers.

Summing up, while the NK literature assumes that growth is an exogenous and independent process with respect to the business cycle, the literature that studies the interplay between growth and business cycle concentrates on the relationship between volatility and growth and disregards the implied optimal monetary policy prescriptions. Thus, to the best of our knowledge we are the first to study the monetary policy implication of this setup.[8]

The paper proceeds as follows. Section 2 describes the model. Section 3 analyzes the Ramsey optimal policy. Section 4 shows results from the search of an optimal operational interest rate rule. Section 5 concludes.

2 A Sticky Price Endogenous Growth Model

The economy is described by a standard New Keynesian model with nominal prices rigidities *à la* Calvo (1983), including an endogenous growth mechanism with serendipitous learning *à la* Romer (1986). There are two sources of uncertainty: the level of total factor productivity and government spending, which is assumed to be fully financed by lump-sum taxes.

2.1 Final Good-Sector

In each period, the final good Y_t is produced by perfectly competitive firms, using the intermediate inputs produced by the intermediate sector, with the standard CES technology: $Y_t = \left[\int_0^1 Y_{j,t}^{(\theta_p-1)/\theta_p} dj\right]^{\theta_p/(\theta_p-1)}$, with $\theta_p > 1$ being the elasticity of substitution between differentiated goods. Taking prices as given, the typical final good producer assembles intermediate good quantities $Y_{j,t}$ to maximize profits, resulting in the usual demand schedule: $Y_{j,t} = (P_{j,t}/P_t)^{-\theta_p} Y_t$. The zero-profit condition of final good producers leads the aggregate price index $P_t = \left(\int_0^1 P_{j,t}^{1-\theta_p} dj\right)^{1/(1-\theta_p)}$.

[7] See for example Schmitt-Grohé and Uribe (2004a), (2007a, 2007b) who, in different models, find that the optimal interest-rate rule features a muted response to output.

[8] In a similar framework Vaona (2012) explores the relationship between inflation and growth.

2.2 Intermediate Good-Sector and Externalities

The market is populated by a continuum of firms acting as monopolistic competitors. We assume that this continuum of intermediate good-producing firms $j \in [0,1]$ employ labor N_t and capital K_t from households to produce Y_t units of the intermediate good using the following technology:

$$Y_{j,t} = A_t K_{j,t}^{1-\alpha} \left(Z_t N_{j,t}\right)^{\alpha}, \quad \alpha \in (0,1), \; A > 0. \tag{1}$$

where Z_t represents an index of knowledge, taken as given by each firm, which is freely available to all firms and which is acquired through learning-by-doing. In particular, we assume $Z_t = K_t$, where $K_t = \int_0^1 K_{j,t} dj$. Following convention, productivity Z_t is taken as given by each firm, so that learning takes the form of a pure externality. The term A_t is an aggregate productivity shock, which follows the following process

$$\log A_t = (1-\rho_A)\log A + \rho_A \log A_{t-1} + \varepsilon_{A,t}, \tag{2}$$

with $0 < \rho_A < 1$ and $\varepsilon_{A,t} \sim i.i.d.N(0, \sigma_A^2)$.

Prices are modeled *à la* Calvo. In each period there is a fixed probability $1 - \xi_p$ that a firm in the intermediate sector can set its optimal price $P_{j,t}^*$ otherwise the price is unchanged.

Let $MC_{j,t}^N$ denote the nominal marginal cost, the cost minimization, taking the nominal wage rate W_t and the rental cost of capital R_t^K as given, yields the standard optimality conditions, $W_t = \alpha MC_{j,t}^N \frac{Y_{j,t}}{N_{j,t}}$ and $R_t^K = (1-\alpha) MC_{j,t}^N \frac{Y_{j,t}}{K_{j,t}}$ which, in turn, imply that real marginal cost, $MC_t = MC_t^N / P_t$, is common to all firms:

$$MC_t = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{1}{A_t Z_t^{\alpha}} \left(\frac{R_t^K}{P_t}\right)^{1-\alpha} \left(\frac{W_t}{P_t}\right)^{\alpha}. \tag{3}$$

The typical firm, able to reset its price at time t, will choose the price P_t^* so as to maximize the expected present discounted value of profits given the demand schedule and the marginal cost MC_t. At the optimum

$$\frac{P_t^*}{P_t} = \frac{\theta_p}{\theta_p - 1} \frac{E_t \sum_{i=0}^{\infty} \xi_p^i Q_{t,t+i} MC_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta_p} Y_{t+i}}{E_t \sum_{i=0}^{\infty} \xi_p^i Q_{t,t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta_p - 1} Y_{t+i}}, \tag{4}$$

where $Q_{t,t+i}$ is the stochastic discount factor used at time t by shareholders to value date $t+i$ profits.

Define the two artificial variables $x_t = E_t \sum_{i=0}^{\infty} \xi_p^i Q_{t,t+i} MC_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta_p} Y_{t+i}$ and $z_t = E_t \sum_{i=0}^{\infty} \xi_p^i Q_{t,t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta_p - 1} Y_{t+i}$ and denote $p_t^* = \frac{P_t^*}{P_t}$. The optimal price equation (4) now reads as follows

$$p_t^* = \frac{\theta_p}{\theta_p - 1} \frac{x_t}{z_t}, \tag{5}$$

where x_t can be written recursively as:

$$x_t = C_t^{-1} Y_t MC_t + \xi_p \beta E_t \pi_{t+1}^{\theta_p} x_{t+1}, \tag{6}$$

while z_t can be written as:

$$z_t = C_t^{-1} Y_t + \xi_p \beta E_t \pi_{t+1}^{\theta_p - 1} z_{t+1}, \tag{7}$$

where $\pi_t = P_t / P_{t-1}$. Finally, the aggregate price level $P_t = \left(\int_0^1 P_{j,t}^{1-\theta_p} dj \right)^{1/(1-\theta_p)}$ evolves according to $P_t = \left[\xi_p P_{t-1}^{1-\theta_p} + \left(1 - \xi_p\right) P_t^{*1-\theta_p} \right]^{1/(1-\theta_p)}$, that is to say that the price level is just a weighted average of the last period's price level and the price set by firms adjusting in the current period. This equation can be rewritten as follows:

$$1 = \xi_p \pi_t^{\theta_p - 1} + \left(1 - \xi_p\right) \left(p_t^*\right)^{1-\theta_p}. \tag{8}$$

2.3 Households

The representative household maximizes the following lifetime utility subject to a sequence of flow budget constraints:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \mu_n \frac{N_t^{1+\phi}}{1+\phi} \right), \ \phi, \ \mu_n > 0 \text{ and } \beta < 1, \tag{9}$$

$$P_t C_t + R_t^{-1} B_{t+1} = B_t + W_t N_t + D_t + R_t^K K_t - P_t I_t - T_t, \tag{10}$$

where C_t is consumption and N_t labor hours at time t, K_t is physical capital, I_t denotes investments and B_{t+1} represents purchases of riskless one-period bonds, and paying one unit of the numéraire in the following period $t+1$, while B_t is the quantity of bonds carried over from period $t-1$. R_t is the gross nominal return on riskless bonds purchased in period t, R_t^K is the gross nominal return on capital, T_t denotes lump-sum taxation and D_t are dividends from ownership of firms. Physical capital accumulates according to:

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{11}$$

where δ is the depreciation rate of capital. The first-order conditions for the consumer's problem can be written as:

$$\frac{1}{R_t} = E_t Q_{t,t+1}, \tag{12}$$

$$\frac{W_t}{P_t} = \mu_n C_t N_t^{\phi}, \tag{13}$$

$$C_t^{-1} = \beta E_t C_{t+1}^{-1} \left(\tilde{R}_{t+1}^k + 1 - \delta \right), \tag{14}$$

where $Q_{t,t+1} = \beta \frac{C_t P_t}{C_{t+1} P_{t+1}}$ is the stochastic discount factor for nominal payoffs and $\tilde{R}_{t+1}^k = R_{t+1}^k / P_{t+1}$.

2.4 Market Clearing

In equilibrium factor and good markets clear, hence the following conditions are satisfied for all t: $N_t = \int_0^1 N_{j,t} dj$, $K_t = \int_0^1 K_{j,t} dj$ and $Y_t D_{p,t} = \int_0^1 Y_{j,t}$ where $D_{p,t} = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_p} dj$ is a measure of price dispersion. Using (1) aggregate production is found to be:

$$Y_t = A_t K_t N_t^{\alpha} \left(D_{p,t}\right)^{-1}, \tag{15}$$

where it is easy to see that $D_{p,t}$ evolves according to a non-linear first-order difference equation:

$$D_{p,t} = \left(1 - \xi_p\right) p_t^{*-\theta_p} + \xi_p \pi_t^{\theta_p} D_{p,t-1}. \tag{16}$$

Finally, the following aggregate resource constraint must hold:

$$Y_t = C_t + I_t + G_t, \tag{17}$$

where G_t is public consumption, fully financed by lump-sum taxation T_t, and, on balanced growth path, is assumed to evolve as a constant fraction of output.

2.5 Stationary Competitive Equilibrium

In this economy a number of variables, such as output, consumption etc. will not be stationary along the balanced-growth path. We therefore perform a change of variables, so as to obtain a set of equilibrium conditions that involve only stationary variables. We note that non stationary variables at time t are cointegrated with K_t, while the same variables at time $t+1$ are cointegrated with K_{t+1}. We divide variables by the appropriate cointegrating factor and denote the corresponding stationary variables with lowercase letters. Equations (5), (8), (16), (2), (28) are already expressed in terms of stationary variables.

Capital and labor demands are now expressed as

$$\tilde{R}_t^K = (1-\alpha) MC_t y_t, \tag{18}$$

$$w_t = \alpha MC_t \frac{y_t}{N_t}, \tag{19}$$

where $y = Y/K$ and $w = W/PK$. In terms of stationary variables the price related equations (6) and (7) are

$$x_t = c_t^{-1} y_t MC_t + \xi_p \beta E_t \pi_{t+1}^{\theta_p} x_{t+1}, \tag{20}$$

$$z_t = c_t^{-1} y_t + \xi_p \beta E_t \pi_{t+1}^{\theta_p - 1} z_{t+1}. \tag{21}$$

The Euler equation (12) can be expressed as

$$c_t^{-1} = E_t \beta R_t \left(c_{t+1} g_{k,t+1}\right)^{-1} \frac{1}{\pi_{t+1}}, \tag{22}$$

where $c = C/K$ and $g_{k,t+1} = K_{t+1}/K_t$. It should be noted that with growth the economy displays a lower marginal rate of intertemporal substitution for consumption, which, in turn, implies a lower effective discount factor.

The labor supply (13) can be written as

$$w_t = \mu_n c_t N_t^{\phi}. \tag{23}$$

The capital Euler equation (14) becomes:

$$c_t^{-1} = E_t \beta \left(c_{t+1} g_{k,t+1}\right)^{-1} \left(\tilde{R}_{t+1}^K + 1 - \delta\right). \tag{24}$$

The capital accumulation equation (11) becomes

$$g_{k,t+1} = 1 - \delta + i_t, \tag{25}$$

where $i = I/K$.

The production function (15) is simply

$$y_t = A_t N_t^{\alpha} \left(D_{p,t}\right)^{-1}. \tag{26}$$

Finally, the resource constraint of the economy (17) in stationary terms is

$$y_t = c_t + i_t + g_t, \tag{27}$$

where $g_t = G_t/K_t$ and evolves as

$$\log g_t = (1 - \rho_G) \log g + \rho_G \log g_{t-1} + \varepsilon_{G,t}, \tag{28}$$

where $0 < \rho_G < 1$ and $\varepsilon_t^G \sim i.i.d. N(0, \sigma_G^2)$.

The competitive equilibrium of the economy under study can now be formally defined.

Definition 1: *A stationary competitive equilibrium is a sequence of allocations and prices* $\{c_t,\, i_t,\, g_{k,t+1},\, N_t,\, y_t,\, \tilde{R}_t^K,\, MC_t,\, w_t,\, \pi_t,\, D_{p,t},\, p_t^*,\, x_t,\, z_t\}_{t=0}^{\infty}$ *that remain bounded in some neighborhood around the deterministic steady state and satisfy equations (5), (8), (16), (18)-(27), given a sequence of nominal interest rate* $\{R_t\}_{t=0}^{\infty}$*, initial value for* $D_{p,t-1}$ *and a set of exogenous stochastic processes* $\{A_t, g_t\}_{t=0}^{\infty}$.

2.5.1 Inefficiencies of the Competitive Equilibrium

The competitive economy considered so far is distorted. In particular, it features three sources of inefficiency providing a rationale for the conduct of monetary policy: (i) monopolistic competition in the intermediate goods sector; (ii) nominal rigidities due to staggered prices introduced *à la* Calvo (1983); (iii) the presence of knowledge spillovers which are external to each firm.

The first two distortions are the ones which characterize the basic NK model and act as follows. Monopolistic competition in the intermediate goods sector generates an *average markup*, which lowers output with respect to the competitive economy. Nominal rigidities due to staggered prices generates price dispersion which, in turn, results in an inefficiency loss in aggregate production since the higher is price dispersion the more inputs are needed to produce a given level of output.[9] It should be noted that in this AK setting, since growth is due to external learning, a higher (lower) level of economic activity leads to higher (lower) growth. That is why eliminating the first two distortions has also positive effects on growth.[10]

The third additional source of inefficiency differentiates the present model from a standard NK model, i.e. the presence of *knowledge spillovers* which are external to each firm. In these circumstances capital accumulation is below its social optimum value since agents do not price the role of capital stock in increasing productivity. Hence, although, the aggregate production function has constant returns to scale in capital, due to knowledge spillovers, firms will accumulate capital as if they are facing a production function with decreasing returns to scale and therefore investments will be sub-optimal and the economy will grow at a lower level than the optimal one.

In this context the decentralized equilibrium is Pareto suboptimal and the economy grows at a lower rate than under the allocation that would maximize the representative household's lifetime utility.

2.6 Calibration

Starting from the stationary model it is then possible to compute the deterministic steady state of the transformed model around which the model is approximated and solved numerically.[11] We set the benchmark parameters in line with the existing literature.

[9] Schmitt-Grohé and Uribe (2007a) show that price dispersion is bounded below at one, so that it is always costly in terms of aggregate output. The intuition is that price dispersion causes firms, despite being symmetric, charge different prices and thus produce different levels of output. This, in turn, decreases the level of aggregate output by Jensen inequality because the elasticity of substitution among goods is larger than one (see Ascari 2004, King and Wolman 1998, Graham and Snower 2004).

[10] Further notice that the marginal productivity of capital, $(1-\alpha) MC_t A_t N_t^{\alpha} (D_{p,t})^{-1}$, in the absence of these two distortions, would turn out to be $(1-\alpha) A_t N_t^{\alpha}$.

[11] All the simulation results have been obtained by following the 'pure' perturbation method devised by Schmitt-Grohé and Uribe (2004c). In particular, the results on the optimal simple rules have been

Time is measured in quarters.

The discount factor β is set to 0.99, so that the annual interest rate is equal to 4 percent. The initial steady state inflation rate is set equal to 4% at annual level. In our calibration we opt to set the Frisch elasticity to the upper end of the microdata estimates ranging from 0.05 to 0.5, hence ϕ is equal to 2 [12]. The parameter μ_n on labor disutility is calibrated to get a steady state value of labor hours equal to 1/3.[13] The price elasticity θ_p is set equal to 6 and the probability that firms do not revise prices ξ_p is set equal to 0.75. Labor return to scale α is set equal to 2/3. Finally, capital depreciation rate δ is 0.025. We calibrate the remaining parameters to have $C/Y = 0.65$ in steady state and a quarterly growth rate of output of 0.5% along the BGP. This implies that in steady state the public consumption - capital ratio g is equal to 0.021, while the output-capital ratio y is 0.1430, so that G/Y=0.1446.

Similarly to Schmitt-Grohé and Uribe (2007a) the persistence of the technology shock is $\rho_a = 0.8556$, while that of the government spending shock is $\rho_g = 0.87$. The standard deviations of productivity and of the government purchases processes are set equal to $\sigma_a = 0.0064$ and $\sigma_g = 0.016$, respectively.

3 Ramsey Monetary Policy

The Ramsey optimal policy is determined by the central bank which maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy. The Ramsey approach allows to study the optimal policy around a distorted steady state, as it is in our model.[14] We assume that *ex-ante commitment* is feasible. In most NK models it is not possible to combine all constraints in a single implementability constraint, thus, as common in the literature, we follow a hybrid approach in which the competitive equilibrium conditions are summarized via a minimal set of equations. Starting from the optimality conditions for households and firms and the resource constraint of the economy, namely (5), (8), (16), (18)-(27), the number of constraints to the Ramsey planner's optimal problem can be reduced by substitution so to have:

$$\beta E_t \frac{1}{c_{t+1}} \left[(1-\alpha) \frac{\mu_n N_{t+1}^{\phi+1} c_{t+1}}{\alpha} + 1 - \delta \right] - \frac{g_{k,t+1}}{c_t} = 0, \tag{29}$$

$$c_t + g_{k,t+1} + g_t - A_t N_t^{\alpha} \left(D_{p,t} \right)^{-1} - (1-\delta) = 0, \tag{30}$$

obtained adapting the codes developed by Faia (2008a) and Faia and Rossi (2012).

[12] On this see Rotemberg and Woodford (1999) and the original microeconometric papers by MaCurdy (1981) and Altonji (1986).

[13] Sensitivity analysis has been done on alternative preference parameters and results are qualitatively unchanged.

[14] See Khan et al. (2003), Schmitt-Grohé and Uribe (2007a), Faia (2009) for a discussion on welfare analysis with a distorted steady state.

$$D_{p,t} - (1-\xi_p)\left(\frac{\theta_p}{\theta_p - 1}\frac{x_t}{z_t}\right)^{-\theta_p} - \xi_p \pi_t^{\theta_p} D_{p,t-1} = 0, \tag{31}$$

$$\xi_p \pi_t^{\theta_p - 1} - 1 + (1-\xi_p)\left(\frac{\theta_p}{\theta_p - 1}\frac{x_t}{z_t}\right)^{1-\theta_p} = 0, \tag{32}$$

$$x_t - \frac{\mu_n N_t^{\phi+1}}{\alpha} - \xi_p \beta E_t \pi_{t+1}^{\theta_p} x_{t+1} = 0, \tag{33}$$

$$\xi_p \beta E_t \pi_{t+1}^{\theta_p - 1} z_{t+1} - z_t + c_t^{-1} A_t N_t^{\alpha} (D_{p,t})^{-1} = 0. \tag{34}$$

In addition it should be noted that since public consumption is fully financed by lump-sum taxes, the government budget constraint does not appear in the implementability constraints for the Ramsey planner. Finally, by noticing that in the absence of monetary frictions the nominal interest rate only enters into the consumption Euler equation (22), this last condition can be omitted from the set of constraints. Without monetary frictions the solution to the Ramsey problem consists of a real equilibrium which is determined for given nominal interest rate (see e.g. Faia 2009, Faia and Rossi 2013). Putting it differently, R_t is set so as to always satisfy (22), given the outcome of the Ramsey plan.[15] In this context, the central bank chooses the policy instrument, namely the inflation rate, to implement the optimal allocation obtained as solution to the Ramsey problem.

Let $\{\lambda_{1,t},\ \lambda_{2,t},\ \lambda_{3,t},\ \lambda_{4,t},\ \lambda_{5,t},\ \lambda_{6,t}\}$ represent sequences of the Lagrange multipliers on the constraints (29), (30), (31), (32), (33), (34), respectively. Given an initial value for the price dispersion, $D_{p,t-1}$, and a set of exogenous stochastic processes for productivity and public consumption, $\{A_t, g_t\}_{t=0}^{\infty}$, the allocations plans for the control variables $\mathbf{d}_t \equiv \{c_t,\ g_{k,t+1},\ N_t,\ \pi_t, D_{p,t},\ x_t,\ z_t\}_{t=0}^{\infty}$ and for the co-state variables $\mathbf{\Lambda}_t \equiv \{\lambda_{1,t},\ \lambda_{2,t},\ \lambda_{3,t},\ \lambda_{4,t},\ \lambda_{5,t},\ \lambda_{6,t}\}_{t=0}^{\infty}$ represent an optimal allocation if they solve the following maximization problem:

$$Min_{\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}} Max_{\{\mathbf{d}_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \mu_n \frac{N_t^{1+\phi}}{1+\phi} + \frac{\beta}{1-\beta} \log g_{k,t+1} \right), \tag{35}$$

subject to constraints (29)-(34), where the discounted sum of utilities of all agents has been expressed in stationary terms (see the Appendix for the full derivation). Since the above maximization problem exhibits forward-looking constraints is intrinsically non-recursive (see Kydland and Prescott 1980). As common practice in the literature (i.e. Khan et al. 2003), we circumvent this problem by augmenting the policy problem with a full set of lagged multipliers, corresponding to the constraints exhibiting future expectations of control variables.

[15] The Lagrange multiplier associated to this last constraint, in fact, would always be equal to zero.

The augmented Lagrangian for the optimal policy problem then reads as follows:

$$
\begin{aligned}
& Min_{\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}} Max_{\{\mathbf{d}_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t E_t \left[\left(\log c_t - \mu_n \frac{N_t^{1+\phi}}{1+\phi} + \frac{\beta}{1-\beta} \log g_{k,t+1} \right) + \right.\right. \\
& + \lambda_{1,t-1} \frac{1}{c_t} \left((1-\alpha) \frac{\mu_n N_t^{\phi+1} c_t}{\alpha} + 1 - \delta \right) - \lambda_{1,t} \frac{g_{k,t+1}}{c_t} + \\
& + \lambda_{2,t} \left(c_t + g_{k,t+1} + g_t - A_t N_t^{\alpha} \left(D_{p,t} \right)^{-1} - (1-\delta) \right) + \\
& + \lambda_{3,t} \left(\frac{D_{p,t}}{\pi_t^{\theta_p}} - \left(1 - \xi_p\right) \frac{1}{\pi_t^{\theta_p}} \left(\frac{\theta_p}{\theta_p - 1} \frac{x_t}{z_t} \right)^{-\theta_p} - \xi_p D_{p,t-1} \right) + \\
& + \lambda_{4,t} \left(\xi_p \pi_t^{\theta_p - 1} - 1 + \left(1 - \xi_p\right) \left(\frac{\theta_p}{\theta_p - 1} \frac{x_t}{z_t} \right)^{1-\theta_p} \right) + \\
& + \lambda_{5,t} \left(x_t - \frac{\mu_n N_t^{\phi+1}}{\alpha} \right) - \lambda_{5,t-1} \xi_p \beta \pi_t^{\theta_p} x_t + \\
+ \quad & \left.\left. \lambda_{6,t-1} \xi_p \pi_t^{\theta_p - 1} z_t - \lambda_{6,t} \left(z_t - c_t^{-1} A_t N_t^{\alpha} \left(D_{p,t} \right)^{-1} \right) \right] \right\}.
\end{aligned}
$$

3.1 The Ramsey Optimal Steady State

In what follows we analyze the optimal monetary policy in the long run by looking at the Ramsey optimal steady-state inflation rate. This amounts to computing the *modified golden rule* steady-state inflation, i.e. the steady-state inflation rate obtained imposing steady-state conditions ex post on the first-order conditions of the Ramsey plan.[16] We find that the steady-state inflation rate associated with the Ramsey optimal policy turns out to be zero in an NK economy characterized by endogenous growth and knowledge spillovers. This happens because the central bank chooses the inflation rate that reduces the distortion induced by the monopolistically competitive behavior of the intermediate-goods producers and eliminates the inefficiency resulting from the relative price dispersion, so pushing the steady-state growth rate closer to the efficient one. Specifically, given the parametrization used in the previous Section, under the Ramsey plan, the balanced-growth path growth rate is slightly higher than under the competitive equilibrium and equal to 0.53% at quarterly level, so confirming that zero steady-state inflation is beneficial for growth.[17]

The intuition for this result is straightforward. First, as already pointed out, monopolistic competition in the intermediate goods sector generates an average markup which

[16] The Ramsey steady-state equilibrium was calculated numerically by using an OLS approach, as described by Schmitt-Grohé and Uribe (2012).

[17] The optimality of the zero steady-state inflation turns out to be confirmed also under alternative parametrization. Results are available from the authors upon request.

lowers the level of economic activity and growth. As shown by King and Wolman (1996), with staggered prices the average markup tends to increase with trend inflation, since price resetting forward-looking firms will set higher prices relatively to their current marginal costs, exactly to offset the erosion of markups and of relative prices that trend inflation creates.[18] In other words, higher trend inflation yields a larger average markup, inducing a larger distortion related to the lack of competition in the intermediate good sector and so a lower steady-state level of economic activity. Second, the price dispersion increases rapidly with trend inflation, so that the negative effect of trend inflation on output and growth through this channel is quite powerful (see Ascari 2004 and Yun 2005).

Overall, both the average markup and price dispersion are therefore increasing with trend inflation, generating a negative steady-state relationship between inflation and output and so between inflation and growth. In such circumstances, the Ramsey planner would find it optimal to set the steady-state inflation equal to zero. In addition, in the presence of knowledge spillovers which are external to each firm, a zero steady-state inflation, by reducing the average markup and by eliminating price dispersion implies a higher return on capital and thus higher savings, so pushing the steady-state growth rate closer to its efficient level.

3.2 The Ramsey Optimal Dynamics

Let's now analyze the dynamic properties of the Ramsey plan in a calibrated version of the model. The dynamic responses of the Ramsey plan are computed by taking second-order approximations of the set of first-order conditions around the steady state.[19]

Figure 1 shows the Ramsey optimal impulse response functions to a one percent positive productivity shock for consumption, inflation, employment, output, rate of growth and nominal interest rates. All results are reported as percentage deviations from the steady state, except inflation, growth and nominal interest rates, which are expressed as percentage-point deviations. As in the competitive economy, output and consumption increase. Inflation rate decreases, but the nominal interest rate is above its long-run level, implying a higher real interest rate and so a counter-cyclical policy. Our findings stand in contrast with those of Faia (2008b) who shows that by simply embedding capital into an NK model the optimal Ramsey monetary policy turns out to be pro-cyclical in response to a technology shock.[20]

[18] The average markup, i.e. $\frac{P}{MC^N}$ (MC^N denoting the nominal marginal cost), can be split in two terms: $\frac{P}{MC^N} = \frac{P}{P_i^*}\frac{P_i^*}{MC^N}$. The first term, i.e., $\frac{P}{P_i^*}$, is the "price adjustment gap", i.e. the ratio between the CPI index P and the newly adjusted price P_i^*, and is shown to decrease with steady state inflation. The second term, i.e., $\frac{P_i^*}{MC^N}$, is the "marginal markup" and is shown to increase with steady state inflation. King and Wolman (1996) demonstrate that the "marginal markup" effect generally dominates the "price adjustment gap" effect, while only for extremely low values of trend inflation the opposite is true.

[19] See Schmitt-Grohé and Uribe (2004c, 2004d).

[20] For robustness check we have also solved the present model under the assumption of quadratic cost

To understand the logic of the different behavior of the Ramsey planner in this context, a few remarks are needed. First, as already emphasized, the serendipitous learning mechanism is the source of an additional distortion which characterizes this economy. Firms, in fact, do not internalize the positive externalities deriving from the use of capital, hence the private marginal productivity of capital is lower than that prevailing with firms fully internalizing the beneficial effects of capital utilization.[21] Thus, in order to reach the second-best allocation, the Ramsey planner creates the conditions to generate an increase in the real interest rate. This, in turn, would imply a higher growth rate and would place consumption on a higher path of growth. Second, in a growing economy the effective discount factor is lower than in an economy with no growth, implying that agents discount more the future.[22] This means that, by inducing an increase in the real interest rate, the central bank would lead to a higher growth rate, as well as to higher consumption. Finally, the stationarized welfare function (35) depends on both growth rate and consumption. Thus, a higher growth rate, would imply higher consumption and so higher welfare. Further, from the welfare equation, it is clear that the weight assigned to growth is higher than that assigned to the stationarized level of consumption.

Having said this, it comes as no surprise that the Ramsey monetary policy turns out to be counter-cyclical instead of pro-cyclical in response to a positive technology shock. In these circumstances, in fact, the real interest rate prevailing in the market does not capture the positive externalities of capital utilization, hence consumption is too high during a period of expansion triggered by a positive productivity shock. In addition, in the competitive economy the effective discount factor will be reduced following the higher growth rate, therefore consumers will tend to consume too much instead of exploiting the temporary period of expansion to enhance current and future growth prospects by accumulating more capital and moving to a higher consumption stream. Thus, the Ramsey planner will find it optimal to offset these effects, by creating the conditions that induce households to build up the capital stock during the early phases of the adjustment process, so boosting growth and, ultimately, moving the economy to a higher balanced growth path.

- Figure 1 about here -

of adjusting prices *à la* Rotemberg (1982) as in Faia (2008b). We find that the optimal monetary policy still prescribes a counter-cyclical response to both shocks. These additional results are available in a separate appendix on the authors' webpages.

[21] The private marginal productivity of capital is $\tilde{R}_t^K = (1-\alpha) MC_t y_t$, while with firms fully internalizing the positive externalities we would have $\tilde{R}_t^K = MC_t y_t$.

[22] From the Euler equation (22) we observe, that the effective discount factor is $\frac{\beta}{g_{k,t+1}}$, where β captures the relative weight placed on the future versus today and $\frac{1}{g_{k,t+1}}$ captures the fact that, thanks to economic growth, agents expect to enjoy a higher consumption in the future.

Figure 2 shows impulse response functions to a one percent positive government spending shock. The government spending shock crowds out consumption and investments. The inflation and the nominal interest rate responses are such that the real rate is always positive along the adjustment path. In this context, the optimal monetary policy calls for an increase in the real rate so as to moderate the temporary expansionary effects of aggregate demand on output. Intuitively, following an increase in public consumption (a pure waste in this economy), the welfare-maximizing planner will find it preferable to suffer a short-run decrease of the growth rate, than a sharp increase in the disutility deriving from non-leisure activity. In this case our findings are consistent with Faia (2008b), where following a positive government expenditure shock, the Ramsey planner will find it optimal to increase the nominal interest rate so to reduce consumption and implying a fall in the price level.

- Figure 2 about here -

Summing up, the optimal Ramsey dynamics requires a deviation from price stability in an economy characterized by endogenous growth and knowledge spillovers. The central bank tolerates a moderately negative inflation rate in order to push the short-run economy growth rate toward the efficient one in response to a positive technology, while increases the real interest rate to mitigate the effects of a positive government spending shock on output and labor hours.

Overall, a model with knowledge spillovers, and thus endogenous growth, is characterized by having the capital growth rate into the welfare function. As a consequence, it turns out that a full inflation targeting policy is far from being optimal.

4 Optimal Operational Interest Rate Rules

Despite the Ramsey policy delivers the optimal policy functions in response to shocks, in practice most of the central banks nowadays implement simple feedback interest rate rules. For this reason, we now study the optimal operational interest rate rules. Such a rule is obtained by searching, within the class of Taylor-type rules, for the parameters that maximize households conditional welfare subject to the competitive equilibrium conditions that characterize the model economy. As well explained by Schmitt-Grohé and Uribe (2004a, 2007a,b) and Faia (2008a) among others, this class of rules must satisfy the following four criteria: a) they must be simple, i.e. they must involve only observable variables; b) they have to guarantee the uniqueness of the rational expectation equilibrium; c) they must be optimal, i.e. they have to maximize the expected life-time utility of the representative agent; d) they must respect the zero bound on nominal interest rates.

Following Schmitt-Grohé and Uribe (2004a, 2007a,b) we focus on the *conditional* expected discounted utility of the representative agent, that is:[23]

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \mu_n \frac{N_t^{1+\phi}}{1+\phi} \right), \tag{36}$$

however, in a model with endogenous growth the value function (36) needs to be stationarized. Thus, after some algebra we find the following expression:

$$\upsilon_t = \log c_t - \mu_n \frac{N_t^{1+\phi}}{1+\phi} + \frac{\beta}{1-\beta} \log g_{k,t+1} + \beta E_t \upsilon_{t+1}. \tag{37}$$

where $\upsilon_t = V_t - \frac{1}{1-\beta} \ln (K_t)$.

The analysis of the optimal rules and the welfare comparison with ad-hoc rules is done based on the Taylor-type class of rules

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\phi_r} \left[\left(\frac{\pi_{t+i}}{\pi} \right)^{\phi_\pi} \left(\frac{y_{t+i}}{y} \right)^{\phi_y} \right]^{1-\phi_r} \quad \text{with } i = 0, +1, -1, \tag{38}$$

where π is the deterministic balanced growth path value of π_t, y is the deterministic BGP value of $y_t = Y_t/K_t$. R is the deterministic BGP value of R_t and ϕ_r, ϕ_π, ϕ_y are policy parameters.

The central bank searches for the optimal rule by maximizing the welfare (37), subject to the constraints represented by the competitive economy relations. Numerically, the search is conducted over the parameter space given by $\{\phi_\pi, \phi_y, \phi_r\}$. The parameter i in the Taylor rule is alternatively set to $i = 0$ for contemporary rules, $i = -1$ for backward-looking rules and finally to $i = 1$ for forward-looking rules. We resort to constrained optimal rules, that is we restrict the grid-search to consider only empirically relevant values for the policy parameters ϕ_π and ϕ_y : we search for ϕ_π restricted to lie in the interval $[1, 3]$ and for ϕ_y in $[0, 2]$ with a step of size 0.0625 and for ϕ_r restricted in $[0, 0.9]$ with a step of size 0.1. As in Schmitt-Grohé and Uribe (2007a) we approximate the non-negativity constraint on the nominal interest rate by requiring that a rule must induce a low volatility of the nominal interest rate around its target level, that is we impose the condition: $2\sigma_R < R$, where σ_R denotes the standard deviation of the nominal interest rate. Table 1 summarizes the results.

[23]Specifically, the conditional measure of welfare assumes an initial state of the economy and allows to take into account the transitional effects from that initial condition to the stochastic steady state implied by the policy rule adopted by the monetary authorities. As commonly assumed, the initial state of the economy coincides with the deterministic steady state.

- Table 1 about here-

The optimal operational rule takes the form:

$$\ln\left(\frac{R_t}{R}\right) = 1.3125 \ln\left(\frac{\pi_{t-1}}{\pi}\right) + 0.9375 \ln\left(\frac{y_{t-1}}{y}\right). \tag{39}$$

Thus, optimized interest-rate rule: i) requires a vigorous response to output and a quite aggressive reaction to inflation; ii) is a backward-looking rule. These results can be explained as follows.

First, it is worth noticing the following. We find that the coefficient attached to past output is positive and strongly different from zero. The coefficient relative to the response to past inflation is, instead, higher than 1, but not too strong. From this point of view this result differs from what is generally found in the literature, where inflation targeting policies tend to be prevail.[24] Intuitively, a rule such as (39) entails a strong counter-cyclical component, which is in line with that implied by the Ramsey monetary policy, where we observe an increase in the real interest rate in response to positive demand and supply shocks. The optimal rule must be designed so as to balance the effects of shocks' uncertainty on the main macrovariables, accounting for the benefits from faster growth. To understand the optimality of a counter-cyclical behavior, for the sake of simplicity, let us consider each shock in turn. An increase in public consumption boosts output and inflation, but tends to crowd out both private consumption and investment, so diminishing current and future growth, and future consumption as well. In these circumstances, the positive inflation and output coefficients of the optimal rule work in the same direction and the real interest rate increases so as to moderate the inflationary pressure and re-establish the incentives to invest more. Putting it differently, in the face of public consumption shocks it is possible to counteract by trying to increase savings by implementing a counter-cyclical policy and by stabilizing inflation variability, so moderating the upward pressure on the markups. Following a positive productivity shock, as already argued, the existence of positive externalities in the production function calls for an increase in the real interest rate to promote capital accumulation and enhance current and future growth prospects.

Second, the optimal rule is backward looking, implying that an inertial response of the nominal interest rate is required to maximize welfare. According to rule (39), in fact, only past conditions are taken into account in choosing the current policy setting. In addition, the inertial behavior of the interest rate turns out to be necessary for every rule considered. Indeed, as shown in Table 1, the optimal operational rule exhibits a strong level of inertia in interest rates and a muted response to output when the feedback part of the rule is

[24]See for example Schmitt-Grohé and Uribe (2007b) who, in a medium scale model, find that the optimal interest-rate rule responds to current price and wage inflation, while it is mostly mute in output, and implies only moderate inertia. In Schmitt-Grohé and Uribe (2007a, 2004a) optimized policy rules feature muted response to output and no inertia.

current-looking or forward-looking. Thus, the optimal operational interests rules require either to target past inflation and past output, or a high response to past interest rate when the policy is current-looking or forward-looking.[25] Putting it differently, it is then desirable that agents be able to fully anticipate the central bank policy conduct one period earlier. This history dependent attitude enables the monetary authority to manage expectations in a way that furthers its stabilization target.[26] As a matter of fact in a forward-looking model, in which agents expectations about future policy are one of the determinants of current outcomes, the reduced uncertainty of the monetary policy conduct itself affects the decision of firms resetting their prices, which will tend to set lower markups so implying an augmented level of economic activity and higher growth. Notably, in the face of higher uncertainty firms able to reset their price will tend to set a higher markup than that prevailing in a deterministic context. The existence of nominal rigidities with imperfect competition is, in fact, conducive of a negative relationship between uncertainty and growth, since higher uncertainty tends to boost average markup.[27] But why should higher uncertainty lead to higher markups? Intuitively, the Calvo's pricing mechanism implies that producers resetting their prices will choose a price that is a positive function of the weighted average of current and expected future nominal marginal costs. Diminishing marginal productivity of labor implies that the nominal marginal cost is a convex function of labor inputs. By Jensen's inequality this, in turn, suggests that a higher variability in labor inputs due to uncertainty, raises average nominal marginal costs and so increases the price set by firms, implying that a higher markup will prevail in the economy. Overall, a more inertial interest rate rule implies a lower degree of uncertainty in the economy and thus higher growth.

Finally, as argued by many authors, both the current and the forward-looking rule are not truly operational. If on the one hand, the forward-looking rules require information on future inflation expectation, which is not directly observable, on the other hand the current rule requires information on the current value of inflation and output which are not in the information set of the central bank. Thus, according to these critics, a rule is truly operational only when is based on past information. Thus, we can claim that the backward-looking rule implied by our model with endogenous growth is not only the optimal one, but it is indeed the truly operational rule.

[25] As a matter of fact, the higher the coefficient ϕ_r the higher the weight assigned to past events, the higher the long-run response to shocks and the more expectations are stabilized.

[26] On this point see Woodford 2003, chapter 7.

[27] On the effect that uncertainty on pricing decisions and on the consequently detrimental effect on long-run growth, see Annicchiarico and Pelloni (2013) and also Annicchiarico et al. (2011b).

5 Conclusions

We consider an NK model characterized by endogenous growth with serendipitous learning *à la* Romer, and nominal rigidities due to staggered price *à la* Calvo. An additional source of inefficiency differentiates our model from the standard NK model, i.e. knowledge spillovers which are external to each firm. The decentralized equilibrium is Pareto suboptimal and the economy grows at a lower rate than under the allocation that would maximize the representative household's lifetime utility. We show that despite the optimal long-run value of inflation is zero, the Ramsey dynamics requires deviation from full inflation targeting in response to technology and government spending shocks. However, the intensity of the reaction crucially depends on the source of fluctuations. Following a positive technology shock the central bank tolerates moderate deviations of the inflation rate below its optimal steady state coupled with a higher nominal rate in order to foster savings and push up the short-run economy growth rate. In response to a positive government shock, optimality calls for an increase in the real interest rate, so as to moderate the effects of the expansionary policy. The optimal operational monetary rule is found to be backward-looking, featuring a strong response to output deviations and a mild reaction to inflation movements. In general, we find that the inertial behavior of the interest rate turns out to be necessary for every rule considered. This history dependent attitude enables the Central Bank to steer expectations in a way that facilitates its stabilization target, because of the reduced uncertainty. These results differ from what is generally found in the literature.

Overall, we find that macroeconomic stabilization policy must explicitly consider the additional transmission channel introduced by an endogenous growth mechanism. In this sense, our analysis provides a further step towards the understanding of the non-trivial interconnections between macroeconomic fluctuations and growth. Therefore, we believe that the NK literature cannot disregard the additional transmission channel introduced by endogenous growth.

The analysis of the present paper has been deliberately restricted to the analysis of monetary policy in the context of a very simple endogenous growth model. We argue that future research should be oriented to explore in more depth these issues considering different and more realistic growth models as well as the implications for both monetary and fiscal policy.

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Appendix

Welfare Measure

The welfare of the typical individual (36) can be written in recursive form as:

$$V_t = \log C_t - \mu_n \frac{{N_t}^{1+\phi}}{1+\phi} + \beta E_t V_{t+1}. \tag{40}$$

By adding and subtracting $\frac{1}{1-\beta}\log K_t$ and $\frac{\beta}{1-\beta}\log K_{t+1}$ we get

$$\begin{aligned} V_t \;\; = \;\; & \log C_t - \mu_n \frac{{N_t}^{1+\phi}}{1+\phi} + \\ & - \log K_t + \frac{1}{1-\beta}\log K_t - \frac{\beta}{1-\beta}\log K_t + \\ & + \frac{\beta}{1-\beta}\log K_{t+1} - \frac{\beta}{1-\beta}\log K_{t+1} + \beta E_t V_{t+1}, \end{aligned} \tag{41}$$

where we have used the fact that $\frac{1}{1-\beta}\log K_t = \log K_t + \frac{\beta}{1-\beta}\log K_t$. Collecting terms and defining $\upsilon_t = V_t - \frac{1}{1-\beta}\ln K_t$ yield (37) which can be also expressed as:

$$\upsilon_t = E_t \sum_{j=0}^{\infty} \beta^j \left(\log c_{t+j} - \mu_n \frac{{N_{t+j}}^{1+\phi}}{1+\phi} + \frac{\beta}{1-\beta}\log g_{k,t+1+j} \right). \tag{42}$$

Figure 1: Dynamic responses to a 1% increase in productivity under Ramsey monetary policy

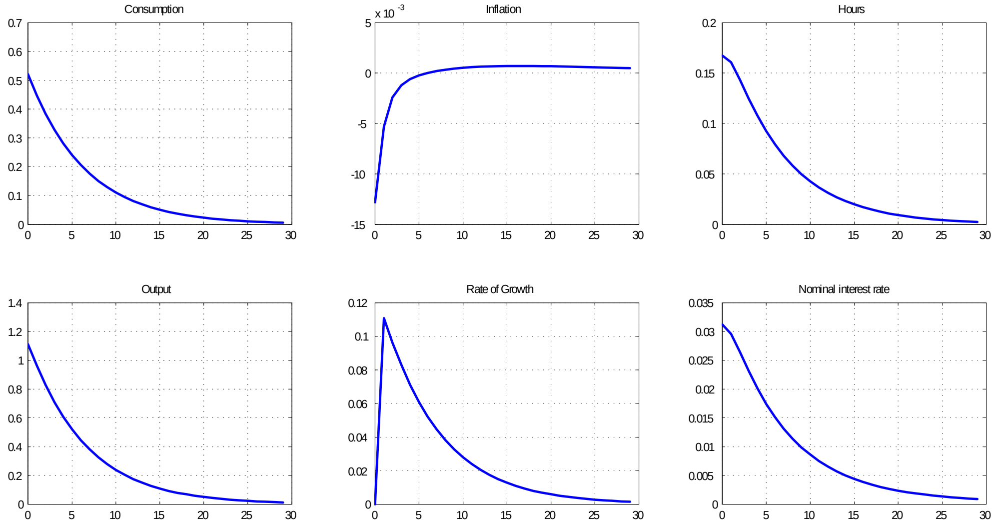

Figure 2: Dynamic responses to a 1% increase in public spending under Ramsey monetary policy

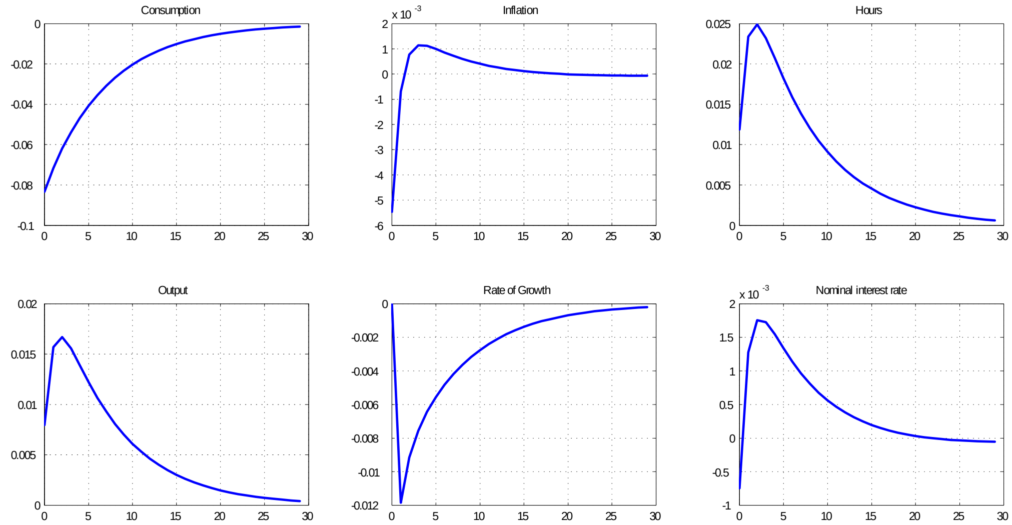

Table 1: Optimal Monetary Policy Rules

	ϕ_π	ϕ_y	ϕ_r	**Conditional Welfare**
Ramsey Policy	—	—	—	-217.6216
Optimal Rules				
Current looking	1.4375	0.0625	0.9000	-221.3196
Backward looking	1.3125	0.9375	0	-217.6375
Forward looking	1.3125	0.0625	0.70000	-220.0160