Climate Policies, Macroprudential Regulation, and the Welfare Cost of Business Cycles*

Barbara Annicchiarico† Marco Carli‡ Francesca Diluiso§

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Abstract

We compare the performance of a carbon tax and a cap-and-trade scheme in a dynamic stochastic general equilibrium model that includes an environmental externality and agency problems associated with financial intermediation. Heterogeneous polluting firms purchase capital by combining their resources with loans from banks and are hit by idiosyncratic shocks that can lead them to default. We find that financial market distortions strongly affect the performance of climate policy throughout the business cycle. The welfare cost of business cycles is substantially lower under a cap-and-trade system than under a carbon tax if financial frictions are stringent, firm leverage is high, and agents are sufficiently risk-averse. The difference in welfare costs shrinks significantly in the presence of simple macroprudential policy rules that weaken the strength of financial market distortions. These policies can go a long way in smoothing business cycle fluctuations and aligning the performance of price and quantity pollution policies, reducing the uncertainty inherent to the government's chosen climate policy tool.

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†Università degli Studi di Roma "Tor Vergata", Dipartimento di Economia e Finanza. Email: barbara.annicchiarico@uniroma2.it

‡Università degli Studi di Roma "Tor Vergata", Dipartimento di Economia e Finanza. E-mail: marco.carli@alumni.uniroma2.eu

§Corresponding author: Bank of England. E-mail: francesca.diluiso@bankofengland.co.uk

1 Introduction

There is a wide consensus among economists and policy analysts on the need for dramatic reductions in anthropogenic greenhouse gas (GHG) emissions to limit disastrous climate change. Yet, it is less clear which policies would best serve this purpose. Carbon pricing is seen by many economists as a cost-effective policy tool that governments should use as part of their broader climate strategy (e.g. Gugler et al., 2021 and OECD, 2021). Nevertheless, when choosing between carbon taxes and cap-and-trade schemes, the two main ways of pricing carbon, there are trade-offs that policymakers need to take into account. This paper primarily compares these two policies from a business cycle fluctuation standpoint. So far, at least in terms of the number of policy initiatives, neither of these two ways of pricing carbon appears to have prevailed. According to World Bank (2023), carbon pricing instruments cover approximately 23% of GHG emissions. As of April 2023, worldwide, there are 73 carbon pricing instruments in operation. This includes 37 carbon taxes (covering around 6% of global GHG emissions) and 36 emission trading systems (18% of GHG emissions).

On the academic side, dating back to Weitzman (1974), the "prices versus quantity" literature has focused on the problem confronting the environmental regulator: whether it would be better to control pollution by pricing emissions with a tax or by fixing a quantity target through a cap-and-trade scheme.[1] This literature shows that the presence of uncertainty can change the relative performance of these instruments, making the policy choice between the two relevant. Although most studies focus on uncertainty related to marginal costs and benefits of environmental policies, recent contributions have looked at the interplay between business cycle uncertainty and climate policy (see, e.g., Annicchiarico and Di Dio, 2015 and Annicchiarico et al., 2022b).

Policies to limit emissions change the way the economy responds to exogenous disturbances. A cap-and-trade scheme entails certainty about future emission levels, but implies uncertainty about environmental compliance costs, given the unpredictable trajectory of allowance prices. A carbon tax, instead, limits the uncertainty on compliance costs, but allowing emissions to move procyclically with economic activity implies uncertainty regarding the achievement of the pollution targets. See, e.g., Metcalf (2009), Aldy and Stavins (2012), and Aldy and Armitage (2020). Environmental regulation affects all agents in the economy, both directly and indirectly, changing their incentives and

[1] Since the seminal contribution of Weitzman (1974) the issue of price versus quantity regulation has been thoroughly studied in several papers, such as Stavins (1996), Hoel and Karp (2001, 2002), Newell and Pizer (2003), Kelly (2005), and Karp and Traeger (2018), among others. See Stavins (2020, 2022) for a comprehensive discussion and review of this literature and related policy implications.

their behavior toward uncertainty and shocks. Whether regulators should use prices or quantities as planning instruments depends on the characteristics of the economy under analysis, such as available technologies, preferences, frictions, and market failures. Thus, in the words of Weitzman (1974, p. 479) "there is no basic or universal rationale for a general predisposition toward one control mode or the other".

This paper reconsiders this topic through the lens of a theoretical model in which financial frictions amplify economic fluctuations, and compares the welfare costs of business cycles in a cap-and-trade regime with auctioned allowances to those arising in the economy where environmental policy takes the form of a carbon tax. In particular, we investigate the welfare cost of business cycles under price or quantity environmental regulations, and explore the potential interactions between climate policies, financial frictions, and macroprudential policy. The framework we use is a dynamic stochastic general equilibrium (DSGE) model with pollution, in which business cycle fluctuations are amplified by the existence of a "financial accelerator" mechanism, as modeled in Christiano et al. (2008, 2014). The financial intermediary sector is characterized by an agency problem arising from asymmetric information and monitoring costs, as in the earlier work of Bernanke and Gertler (1989) and Bernanke et al. (1999). At the heart of the model are producers in the capital-intensive sector who borrow from banks, are subject to idiosyncratic productivity shocks, and whose activity generates polluting emissions that negatively affect the economy. These producers are subject to environmental regulation and choose the least-cost combination of emissions abatement costs and policy costs (carbon tax payments or emission allowance purchases). Imperfections in the financial market determine the conditions under which credit is granted and interact with the performance of environmental policies. Business cycle fluctuations are generated by shocks to total factor productivity in the final-good sector and by risk shocks in the capital-intensive polluting sector. When firms are hit by adverse shocks, they may not be able to repay their loans, experience failure, and go bankrupt. In this setting, greater uncertainty leads to an increase in risk premia and expands the size of left-tail events.[2]

Our findings point in the direction of strong interactions between financial frictions and the performance of environmental regulation throughout the business cycle. We find that a cap-and-trade system keeps the economy significantly more stabilized. Under a cap-and-trade policy, since the emission permit price moves procyclically, producers bear higher costs to comply with the environmental regulation during an economic upturn

[2]Risk shocks are disturbances to the cross-sectional dispersion of idiosyncratic shocks hitting polluting firms. Risk shocks and higher uncertainty have been emphasized in the literature as essential drivers of macroeconomic fluctuations. See, for example, Christiano et al. (2014), Segal et al. (2015), Caldara et al. (2016), and Bloom et al. (2018).

and incur lower costs in the face of a recession. As a result, a cap-and-trade scheme works to dampen business cycle fluctuations. Instead, under a carbon tax regime, firms pay a constant fee to pollute, and the (relative) costs of compliance are slightly countercyclical. In this case, firms can take advantage of an economic upturn and expand their production by more than allowed under a cap. At the same time, when there is a recession, polluting producers have to cut their production by more than they would under a quantity restriction because of the increase in their compliance costs.

In contrast to previous contributions, our findings also show a significant difference in the welfare effects of the two market-based policies. A cap-and-trade entails substantially lower welfare costs of business cycles than those observed under a carbon tax. This difference is only partially mitigated when the cap and the carbon tax are allowed to respond optimally to economic fluctuations. The fact that financial markets do not work perfectly and that polluting firms could go bankrupt makes the differences between the two environmental regimes significant. In particular, the higher the credit leverage of polluting firms, the stronger the channel of propagation of shocks exerted by financial effects and the more volatile the economy will be under price regulation. When the exposure of polluting firms to external financing decreases, the channel through which financial effects spread weakens: the dynamics of the two environmental regimes will become more similar, and business cycle welfare costs will be lower.

Finally, we explore the potential of countercyclical macroprudential rules. We find that in this context, enacting a macroprudential policy in the form of countercyclical reserve requirements (see, e.g., Leduc and Natal, 2018) or interest rate subsidies to depositors tends to significantly reduce the welfare cost of business cycles, while also aligning the performance of various environmental regulations.

This paper contributes to an expanding body of literature on the relationship between business cycles and environmental policy in the context of DSGE models.[3] The first contributions to this literature are those of Fischer and Springborn (2011), Heutel (2012), and Angelopoulos et al. (2013) who study pollution policies and optimal responses to the business cycle in environmental variants of the baseline real business cycle (RBC) model.[4] Environmental DSGE models have been extended to include other sources of shocks (e.g., Khan et al. 2019 who explore the effects of news shocks), multiple

[3]These models incorporating environmental characteristics into dynamic stochastic macroeconomic frameworks are also known as environmental DSGE or E-DSGE models, following the terminology introduced by Khan et al. (2019).

[4]For an early discussion on this topic, see Bowen and Stern (2010), while for a review of the literature and a discussion of the policy implications, see Annicchiarico et al. (2022a). For empirical analyses on emission dynamics throughout the business cycle, see Doda (2014) and Klarl (2020).

sectors and sources of energy (e.g., Dissou and Karnizova 2016), nominal rigidities and monetary policy (e.g., Annicchiarico and Di Dio 2015, 2017; Annicchiarico and Diluiso 2019, who explore the interaction between pollution and monetary policy in a closed or open economy), and credit market imperfections (e.g., Carattini et al. 2021, Diluiso et al. 2021, Huang et al. 2021, 2022, Giovanardi and Kaldorf 2023, who focus on transition risk). In particular, Huang et al. (2021, 2022) use a framework with borrowing constraints and endogenous default in the spirit of the costly-state-verification model of Christiano et al. (2008, 2014), like the one we adopt in this paper. The authors study the macro-financial impact of tightening environmental policies, showing how the adoption of more stringent environmental regulations and climate change policies could result in adverse effects on firms, exposing financial institutions and the financial system to potential losses. In this paper, rather than focusing on transition risk, we study the performance of different environmental regulations throughout the economic cycle and look at how macroprudential policy could help reduce the uncertainty that comes with regulations on emissions price or quantity.[5]

Among all previous contributions to this line of research, one of the most relevant to our analysis is that of Fischer and Springborn (2011), who compare the performance of alternative environmental policies (price, quantity, and emission intensity) over the business cycle, showing that quantity regulation has a built-in damping effect on short-run fluctuations. A similar exercise is conducted by Annicchiarico and Di Dio (2015) in the context of a New Keynesian model, where it is shown that the ability of quantity regulation to dampen business-cycle fluctuations is increasing in the degree of nominal rigidities. However, when comparing the welfare performances of quantity and price regulations, both contributions find that the two policies are not significantly different. On the contrary, in their multi-sector environmental DSGE model, Dissou and Karnizova (2016) show that, when energy-related shocks are the main driving force of economic fluctuations, a cap policy is significantly less costly than a tax in terms of welfare, in addition to the fact that a quantity restriction delivers a lower level of macroeconomic volatility. As highlighted above, our findings support the presence of significant differences in the welfare cost of business cycles, underlining the importance of the financial channel, a common missing element in the aforementioned papers.

Going beyond the environmental DSGE literature, by exploring the potential role of macroprudential regulation in shaping the performance of environmental policies over

[5]Compared to other papers looking at the interactions between climate policy and macroprudential regulations, we do not focus on green-biased macroprudential interventions. Macroprudential policy in our paper has not the goal of greening the economy but, in line with its nature, has the goal of improving the stability of the financial system and making it more resilient to shocks.

the business cycle, our paper contributes more broadly to the ongoing debate about the potential role that central banks and financial regulators can play in supporting climate change mitigation policies (e.g., Carney 2015, Rudebusch 2019, Bolton et al. 2020, NGFS 2020a,b). Our results suggest that simple countercyclical financial regulations, designed to stabilize the economy, can indirectly reduce the uncertainty inherent to different climate policies, aligning their performance, and thus broadening the menu of mitigation options.

The remainder of the paper is organized as follows. Section 2 presents the utility-based theoretical framework in which we conduct our analysis. Section 3 describes the calibration of the structural parameters of the model. Section 4 looks at the dynamics of the model and evaluates the welfare cost of business cycles under price and quantity pollution policies. Section 5 analyzes the potential role of macroprudential regulation in affecting the macroeconomic performance of environmental policies. Section 6 concludes the paper.

2 The Model Economy

We introduce environmental characteristics in a framework similar to that developed by Christiano et al. (2008, 2014) incorporating the debt contracting model of Bernanke and Gertler (1989) and Bernanke et al. (1999). The core of our model is at the level of the capital-intensive sector, where intermediate-good firms, differing in their net worth, experience idiosyncratic shocks and, via the production process, generate polluting emissions that harm the overall output of the economy. Emissions can be reduced by incurring additional costs through abatement technology. Before the occurrence of shocks, each intermediate-good firm purchases capital from perfectly competitive capital-good producers using internal financing (net worth) and loans obtained from perfectly competitive banks. The structure of the model allows for the possibility of default. In the case of bankruptcy, after incurring a monitoring cost, banks seize the assets of firms that cannot repay their loans. The supply side of the model is closed by a final-good sector that combines intermediate goods with labor. At the other end of the economy, households enjoy consumption, supply labor and hold bank deposits. Finally, the model features a government that sets environmental policy.

2.1 Households

There are a large number of identical households, each of which owns a large number of intermediate-good firms. The representative household derives utility from consumption C_t and disutility from hours worked H_t. Household preferences are of the following non-additively separable types:

$$U_0 = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t^{\sigma_L} (1 - H_t)^{1-\sigma_L} \right]^{1-\eta}}{1 - \eta} \right\}, \tag{1}$$

where $\mathbb{E}_0$ is the rational expectations operator, $\beta \in (0, 1)$ is the subjective discount factor, $\sigma_L \in (0, 1)$ and $\eta \geq 0$ are the preference parameters.
The period-by-period budget constraint is the following:

$$C_t + B_{t+1}^H \leq W_t H_t + (1 + R_{t-1}) B_t^H + T_t. \tag{2}$$

Household sources of funds are income from labor $W_t H_t$, risk-free deposits B_t^H carried over from $t - 1$ and lump-sum payments T_t that include transfers from firms and the government. These funds are allocated to consumption and savings in the form of new deposits B_{t+1}^H. The rate of return R_{t-1} on deposits is assumed to be preset. In equilibrium, this generates a predetermined return for lenders, resulting in borrowers absorbing all risk.[6] The typical household chooses C_t, H_t and B_{t+1}^H with the objective of maximizing (1) subject to (2).

2.2 Final-Good Producers

The final good Y_t is produced by a representative firm through a combination of the intermediate good X_t purchased at a price r_t^x and the labor provided by households according to the following Cobb-Douglas technology:

$$Y_t = A_t X_t^{\alpha} H_t^{1-\alpha}, \tag{3}$$

where $\alpha \in (0, 1)$ and A_t is a measure of total factor productivity (TFP) that is negatively affected by pollution:

$$A_t = \bar{A}_t (1 - D_t(M_t)), \tag{4}$$

[6]This assumption, together with intermediate-good producers being risk neutral and caring about expected net worth in the following period only, is key in shaping the amplification of shocks in the economy. Carlstrom et al. (2016) and Dmitriev and Hoddenbagh (2017) show that relaxing these assumptions implies an attenuation of the financial accelerator effect of the model.

where $\bar{A}_t$ is an exogenous process subject to shocks and D_t refers to a damage function that depends on cumulative emissions M_t. This function captures the negative externality of pollution that motivates environmental regulation.[7]

2.3 Capital-Good Producers

At the end of each period, t, competitive capital-good producers purchase capital from intermediate-goods producers for the price $Q_{K,t}$, rebuild depreciated capital, and build new capital K_{t+1} with the following technology, with installation costs increasing in the rate of investment growth:

$$K_{t+1} = (1-\delta)K_t + (1 - S(I_t/I_{t-1}))I_t, \tag{5}$$

where $\delta \in (0,1)$ denotes the capital depreciation rate, I_t represents investments, and $S(\bullet)$ is an increasing and convex function such that in steady state, $S = S' = 0$. The new capital stock is then sold for the same price $Q_{K,t}$.

2.4 Intermediate-Good Producers and Banks

The intermediate-good sector of the economy is populated by a mass of heterogeneous firms differing in their net worth and using physical capital as a production input. Firms have different levels of wealth because they experienced idiosyncratic shocks in the past.

After production in period t, the state of a typical intermediate-good producer is summarized by its net worth, $N \geq 0$. We assume that for each value of N there are many producers. Let $f_t(N)$ be the density of producers with a net worth of N, then the total net worth of the economy is:

$$N_{t+1} = \int_0^\infty N f_t(N) dN. \tag{6}$$

Following Christiano et al. (2014), we will focus on the behavior of a typical producer with net worth N and use the superscript N to refer to variables of the firm of this type. At the end of the period t, when the net worth of intermediate-good producers is known, each N-type firm obtains a loan, B^N_{t+1}, from a bank. This loan is then combined with the net worth of the firm to purchase capital goods, K^N_{t+1}, in an anonymous and competitive

[7]Modelling damages in the production function is common practice in the climate economics literature. Via its effect on the production possibilities of the economy, climate change negatively affects welfare. An alternative formulation would be that of modeling damages directly in the utility function. To the extent that we keep the assumption of proportionality between damages and GDP, the two formulations are almost equivalent. For a discussion of this point, see Hassler et al. (2016).

market at a price of $Q_{K,t}$. This implies that at the end of period t the balance sheet of the N-type firm is equal to $Q_{K,t}K^N_{t+1} = N + B^N_{t+1}$, from which we can define a measure of leverage, L^N_t, as follows:

$$L^N_t = \frac{Q_{K,t}K^N_{t+1}}{N} \quad \text{or} \quad L^N_t = \frac{N + B^N_{t+1}}{N}. \tag{7}$$

After purchasing capital, the firms carry out the period $t+1$ production process according to the following linear technology:

$$X^N_{t+1} = \omega K^N_{t+1}, \tag{8}$$

where ω is the level of idiosyncratic productivity. Following Bernanke et al. (1999) and Christiano et al. (2014), it is assumed that ω is a log-normally distributed unit mean random variable that is drawn independently over time and across firms with a cumulative distribution function denoted by $F_t(\omega)$. The realization of the random variable ω is observed by the producer, but can only be detected by the bank if it pays a monitoring cost. Let σ_t be the standard deviation of $\log \omega$. This dispersion is allowed to vary randomly over time, and is the source of risk shocks that determine the extent of the cross-sectional dispersion of the idiosyncratic productivity level. The random variable σ_t then represents the risk to which capital-intensive producers are subject in the model. The idea is that of capturing in a parsimonious way the idiosyncratic risk of enterprises and the extreme dispersion of payoffs observed in the data, across sectors and countries (see, e.g., Hall and Woodward 2010, Michelacci and Schivardi 2013). As we will see below, as a result of adverse circumstances, businesses can experience failures.[8]

In the spirit of the DICE model by Nordhaus (2008) and as in Heutel (2012) and Annicchiarico and Di Dio (2015) among others, we assume that the production process is polluting and that emissions, say E_t, depend on the abatement effort and production:

$$E^N_{t+1} = \chi(1 - \kappa^N_{t+1})X^N_{t+1}, \tag{9}$$

where $\chi > 0$ is a scale parameter and $\kappa^N_{t+1} \in (0, 1)$ is the fraction of emissions abated. Clearly, the level of emissions also depends on the realization of the idiosyncratic shock ω. Abatement activity is assumed to be costly, and total abatement spending is described

[8] In principle, the standard deviation σ_t could be endogenized and associated with exposure to risks arising from climate change-induced natural disasters (i.e., tail risks), as well as to risks associated with technological breakthroughs and ecological transitions that could render certain businesses obsolete. However, this line of research is beyond the scope of the paper and is left for future analysis.

by a cost function that depends on the effort made to reduce emissions and on the level of production, that is, $\theta_1 \left(\kappa^N_{t+1}\right)^{\theta_2} X^N_{t+1}$ where $\theta_1 > 0$ and $\theta_2 > 1$. Firms are subject to environmental policy and face an extra cost, P^E_{t+1}, that must be paid for each emission unit. In each period, producers can choose between incurring abatement costs or paying the regulation price (i.e., purchasing emission permits on the market in the case of quantity regulation or paying a specific tax in the case of price regulation). The optimal abatement choice will eventually be such that:

$$\theta_1\theta_2(\kappa^N_{t+1})^{\theta_2-1} = \chi P^E_{t+1}. \tag{10}$$

The above condition implies that the abatement effort will be equated across intermediate-good firms, regardless of their net worth level and their specific productivity level (i.e., $\kappa^N_{t+1} = \kappa_{t+1}$). At the end of the production process of period $t+1$, productive capital depreciates and firms are left with $(1-\delta)\omega K^N_{t+1}$ units of it. This capital stock is then sold in a competitive market to capital-good producers for the price $Q_{K,t+1}$.

Taking everything into account and recalling that the intermediate good is sold for r^x_{t+1} to final-good producers, in period $t+1$ an intermediate-good producer enjoys a (gross) rate of return $\omega\left(1+R^k_{t+1}\right)$, where:

$$1 + R^k_{t+1} = \frac{r^x_{t+1} + (1-\delta)Q_{K,t+1}}{Q_{K,t}} - \frac{\theta_1\kappa^{\theta_2}_{t+1} + P^E_{t+1}\chi(1-\kappa_{t+1})}{Q_{K,t}}. \tag{11}$$

The first term on the right-hand side measures the returns on capital, while the second one measures environmental regulation compliance costs per unit of capital, that is our measure for relative compliance costs that will come in handy when discussing our results.

As anticipated, firms can self-finance only a portion of their capital stock and rely on external financing to complement their net worth as a funding source. The loan obtained by each producer in period t takes the form of a standard debt contract that specifies Z_{t+1}, as the gross interest rate on the debt, and $\bar{\omega}^N_{t+1}$, as the value of ω that divides intermediate-good producers who cannot repay the interest and principal from those who can repay, that is:

$$\bar{\omega}^N_{t+1}(1+R^k_{t+1})Q_{K,t}K^N_{t+1} = B^N_{t+1}Z^N_{t+1}. \tag{12}$$

Firms experiencing an idiosyncratic shock below the cut-off level $\bar{\omega}^N_{t+1}$ go bankrupt.

Intermediate-good producers value a particular debt contract according to the expected return from operating risky technology over the return from depositing net worth in a bank, that is:

$$
\begin{aligned}
&\frac{\mathbb{E}_t\left\{\int_{\bar{\omega}_{t+1}^N}^{\infty}\left[\omega(1+R_{t+1}^k)Q_{K,t}K_{t+1}^N - B_{t+1}^N Z_{t+1}^N\right]dF_t(\omega)\right\}}{N\left(1+R_t\right)} \\
&= \mathbb{E}_t\left[1-\Gamma_t\left(\bar{\omega}_{t+1}^N\right)\right]\frac{1+R_{t+1}^k}{1+R_t}L_t^N,
\end{aligned}
\tag{13}
$$

where in the second line we have used (7) and (12) to express this expected return as a function of the leverage L_t^N, while $1-\Gamma_t\left(\bar{\omega}_{t+1}^N\right)$ represents the share of average earnings received by producers, with $\Gamma_t\left(\bar{\omega}_{t+1}^N\right) = (1-F_t(\bar{\omega}_{t+1}^N))\bar{\omega}_{t+1}^N + G_t(\bar{\omega}_{t+1}^N)$ and $G_t(\bar{\omega}_{t+1}^N) = \int_0^{\bar{\omega}_{t+1}^N}\omega dF_t(\omega)$. See Appendix A for details.

Banks specialize in lending to intermediate-good producers with specific net worth levels, and each of the identical banks holds a large portfolio of loans that is perfectly diversified across producers. Banks obtain resources by issuing B_{t+1}^N in deposits to households at the predetermined interest rate R_t. Moreover, they monitor intermediate-good producers and collect assets (net of monitoring costs) from those who default, hence the following cash constraint with the free-entry condition holds:

$$
\begin{aligned}
&(1-F_t(\bar{\omega}_{t+1}^N))B_{t+1}^N Z_{t+1}^N + (1-\mu)\int_0^{\bar{\omega}_{t+1}^N}\omega dF_t(\omega)(1+R_{t+1}^k)Q_{K,t}K_{t+1}^N \\
&= B_{t+1}^N\left(1+R_t\right),
\end{aligned}
\tag{14}
$$

where the first term on the left-hand side indicates revenues received from the fraction of firms with $\omega_{t+1} \geq \bar{\omega}_{t+1}^N$, namely those which do not go bankrupt, while the second term measures the revenues obtained from bankrupt firms, with μ denoting the proportion of assets lost for monitoring.[9] The idea is that in situations where a borrower reports a low level of productivity and subsequently defaults on a loan, the bank will bear the costs of monitoring to verify the accuracy of what the borrower reported. However, if the borrower reports a high productivity level and pays back the loan as expected, the bank does not need to incur the monitoring expenses.

Using (13), condition (14) can be re-written in a more compact way as follows

$$
\left[\Gamma_t\left(\bar{\omega}_{t+1}^N\right) - \mu G_t(\bar{\omega}_{t+1}^N)\right]\frac{Q_{K,t}K_{t+1}^N}{B_{t+1}^N}(1+R_{t+1}^k) = 1+R_t. \tag{15}
$$

[9] Note that (14) holds with strict equality since we assume there is free entry, and it also implies that the banks' return on deposits is equal to the predetermined rate. The condition (14) determines the 'menu' of the state-contingent debt of contracts $(\bar{\omega}_{t+1}^N, L_t^N)$ that can be offered in equilibrium.

The above equation highlights the main market failure of the economy, on top of climate change, namely the fact that the risk-free interest rate R_t is proportional to the average and not to the marginal return on production R^k_{t+1}. The equilibrium is inefficient because the marginal return exceeds the average return. As a result, the economy is characterized by too little borrowing, that is, the leverage is too low.

Using the definition of leverage, the previously defined functions, (15) can be rewritten in a more compact way as:

$$\Gamma_t\left(\bar{\omega}^N_{t+1}\right) - \mu G_t(\bar{\omega}^N_{t+1}) = \frac{1+R_t}{1+R^k_{t+1}} \frac{L^N_t - 1}{L^N_t}. \tag{16}$$

Intermediate-good producers choose the debt contract that maximizes their objective (13) among the $(\bar{\omega}^N_{t+1}, L^N_t)$ combinations that satisfy (16). Since the constraint is independent of net worth (which only appears as a constant of proportionality in the objective function), all firms will eventually select the same debt contract that can be represented as $(\bar{\omega}_{t+1}, L_t)$ or equivalently as (Z_{t+1}, L_t), irrespective of their net worth (see Appendix A for more details of the derivations).

Finally, it is assumed that at the end of the period $t+1$, a random fraction $1-\gamma$ the assets of each firm assets are eventually transferred to their household, while the rest remains with the producer. This is because firms are owned by households, which in turn instruct polluting producers to maximize their expected net worth. The higher the level of net worth, the greater the amount of resources transferred to households in each period. In addition, producers receive an exogenous lump-sum transfer from the household, say W^p_{t+1}. Note that here we are implicitly making use of the so-called "large family assumption", according to which each household owns many intermediate-good producing firms so that the net worth that goes to the representative family is a fraction $1-\gamma$ of the average net worth of the economy as a whole.

2.5 Aggregation and the Resource Constraint

At the end of the period t the amount of capital purchased by intermediate-good producers must be equal to the amount produced by capital producers, K_{t+1}:

$$K_{t+1} = \int_0^\infty K^N_{t+1} f_t(N) dN. \tag{17}$$

Recalling (8), the aggregate supply of intermediate goods to be used in the final-good sector immediately follows:

$$X_t = \int_0^\infty \int_0^\infty \omega K_t^N f_t(N) dN dF_t(\omega) = K_t. \tag{18}$$

By the law of large numbers, at the end of the period t, the aggregate profits of all $N-$type intermediate-good producers are $[1 - \Gamma_{t-1}(\bar{\omega}_t)](1 + R_t^k)Q_{K,t-1}K_t^N$, therefore recalling that a fraction $1 - \gamma$ of each producer's net worth is transferred to households as a lump-sum, the aggregate net worth evolves as follows:

$$N_{t+1} = \gamma [1 - \Gamma_{t-1}(\bar{\omega}_t)](1 + R_t^k)Q_{K,t-1}K_t + W_t^p, \tag{19}$$

where W_t^p denotes the amount of lump-sum transfers made by households. The aggregate quantity of debt in period t, say B_{t+1}, is then:

$$B_{t+1} = \int_0^\infty B_{t+1}^N f_t(N) dN = Q_{K,t}K_{t+1} - N_{t+1}. \tag{20}$$

In equilibrium, the total funds supplied to intermediate-good producers must be equal to the deposits held by households, that is, $B_{t+1} = B_{t+1}^H$. The state-contingent interest rate Z_t can be obtained by integrating (12) with the density $f_t(N)$.

Aggregate abatement costs and aggregate emissions immediately follow:

$$\theta_1 \kappa_t^{\theta_2} K_t = \theta_1 \kappa_t^{\theta_2} \int_0^\infty \int_0^\infty \omega K_t^N f_t(N) dN dF_t(\omega), \tag{21}$$

$$E_t = \chi(1 - \kappa_t) \int_0^\infty \int_0^\infty \omega K_t^N f_t(N) dN dF_t(\omega) = \chi(1 - \kappa_t)K_t. \tag{22}$$

Finally, the resource constraint of the economy is as follows:

$$Y_t = I_t + C_t + \theta_1 \kappa_t^{\theta_2} K_t + \mu G_t(\bar{\omega}_t)(1 + R_{t+1}^k)Q_{K,t-1}K_t, \tag{23}$$

where the last term on the right represents the aggregate monitoring costs. The net output, say Y_t^n, then is simply equal to $Y_t - \theta_1 \kappa_t^{\theta_2} K_t - \mu G_t(\bar{\omega}_t)(1 + R_{t+1}^k)Q_{K,t-1}K_t$.

2.6 Pollution, Damage, and Environmental Policy

As seen, production at the intermediate-good level generates emissions. Polluting gases accumulate in a stock M_t according to the following law of motion:

$$M_t - \bar{M} = \sum_{s=0}^{t-T} (1-\delta_M)^s \left(E_{t-s} + E^*_{t-s}\right), \tag{24}$$

where $\bar{M}$ denotes the pre-industrial concentration of pollutants, with industrialization having started at time T, $\delta_M \in (0,1)$ measures the natural decay rate of greenhouse gases in the atmosphere, and E^* refers to rest-of-the-world emissions and is kept constant for simplicity. Similarly to Golosov et al. (2014) and consistently with Nordhaus (2008), the accumulation of polluting emissions negatively affects the total factor productivity through a damage function D_t:

$$1 - D_t(M_t) = \exp\left(-\xi\left(M_t - \bar{M}\right)\right), \tag{25}$$

where $\xi > 0$ is a damage parameter that measures the intensity of the negative environmental externality on production or, analogously, the fraction of output lost for each additional unit of pollutants. Climate change is then a stock externality since it is a function of the accumulated stock of emissions rather than of emissions *per se* at any time.[10] Considering the low decay rate δ_M, marginal damages are negligibly affected by temporary changes in emissions over the business cycle.

The government can implement two alternative environmental policies to control pollution: a carbon tax and a cap-and-trade system in which allowances are auctioned. We look at these two policies because they represent two points of comparison that are often brought up in academic and policy discussions. Under a carbon tax regime, a tax rate per unit of emission is imposed: in this scenario, P^E_t is set constant, say $\bar{P}^E$, and can then be interpreted as a carbon tax. Instead, under a cap-and-trade regime, a cap, say $\bar{E}$, is applied to the total emissions E_t generated by the economy.

Finally, we assume that the fiscal authority runs a balanced budget at all times and that carbon pricing revenues, $P^E_t E_t$, are redistributed to households as lump-sum

[10]Note that this is a parsimonious way to introduce the effects of climate change into the model. Instead of modeling damages in two steps (from carbon stock into temperatures and from temperatures into damages), as e.g. in Nordhaus (2008), it is standard practice in several aggregate models to implicitly assume that climate change is a function of the atmospheric stock of greenhouse emissions. The damage effects are multiplicative as in the DICE model and the exponential specification represents a good approximation of Nordhaus' specifications. See Golosov et al. (2014) for a discussion.

transfers, say Tr.[11]

The equilibrium conditions that describe the model economy are summarized in Appendix B.

3 Calibration

This section describes our calibration strategy. Time is measured in quarters, and the model is calibrated to US data. We partition the model parameters into three categories: standard macroeconomic parameters, parameters related to financial frictions, and parameters associated with environmental externalities and pollution policies. Table 1 summarizes the calibration. In the context of the standard parameters related to the backbone of the macroeconomic model, the discount factor β is set to a value consistent with a real interest rate of 4% per year. In accordance with Christiano et al. (2014), the depreciation rate of capital δ is set to 0.025, while the production parameter α to 0.4. We set the relative risk aversion (RRA) to 2 and the preference parameters $\sigma_L = 0.21$ and $\eta = 5.72$. The implied value for the working time is then 0.17. The level of total factor productivity, net of environmental damage, $\bar{A}$ is set to 1.26, so the steady-state value of production Y is equal to 1.

To calibrate the parameters related to the financial part of the model, we mainly follow Christiano et al. (2014). We set the parameter measuring monitoring cost, μ, to 0.21, while the fraction of net worth transferred to households by intermediate-good producers, $1 - \gamma$, is fixed at 0.035. Finally, the standard deviation of the log of the idiosyncratic shock ω is set to 0.3 to deliver a risk premium of 0.52 percentage points, close to the one observed in US data for the period 1985Q1-2019Q4.[12] The implied leverage ratio, L, is around 2, that is close to the mean leverage ratio observed in the US in the decade before the Covid crisis.[13]

To calibrate the environmental block of the model, we rely mainly on the DICE model by Nordhaus (2018a). We start by anchoring the stock of atmospheric carbon concentration to 891 gigatons of carbon (GtC), the approximated value observed in

[11] This is a standard assumption in the absence of any distortionary taxation. Note that we also abstract from the higher administrative costs that even the simplest cap-and-trade systems may require for implementation.

[12] The quarterly Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on the 10-Year Treasury Constant is 0.58 percentage points. See Federal Reserve Bank of St. Louis https://fred.stlouisfed.org/series/BAA10YM.

[13] See Federal Reserve Bank of St. Louis https://fred.stlouisfed.org/series/TOTDTEUSQ163N, where we observe that total debt to equity is around 100%, that is our B/N. Given our definition of leverage, $L = 1 + B/N$, the implied value for it immediatly follows.

the no-policy scenario in the DICE model in 2020. Knowing that the pre-industrial atmospheric concentration of carbon, $\bar{M}$, is approximately 581 GtC (see, i.e. Golosov et al. 2014), we obtain $M - \bar{M}$. The quarterly decay rate of greenhouse gases δ_M is fixed at 0.0021 to reflect a half-life of carbon in the atmosphere of almost 83 years, consistent with Reilly and Richards (1993). The overall level of emissions immediately follows from (24). To determine E, we use World Bank data for the period 1985-2018 and observe that the average share of worldwide carbon dioxide emissions attributed to the US is around 20%. The emission intensity parameter χ is then implied. Using the simulation value of the DICE model on the fraction of production lost due to environmental damage in 2020, which is 0.002438, we can calculate the parameter of the damage function ξ. The parameter θ_2 of the abatement function is fixed at 2.6 following Nordhaus (2018b), while the scale coefficient θ_1 is normalized to one so that the total abatement costs as a fraction of the output are around 0.0019% of GDP. The steady-state price of carbon, P^E, is fixed so that the environmental tax revenues as a share of output are 0.7% in steady state, which is a level consistent with the environmental tax revenues of the US in 2017, according to OECD data.[14] Note that the non-stochastic steady state associated with each pollution policy regime is the same. Lastly, the business cycle is caused by shocks to total factor productivity (TFP) and the standard deviation of idiosyncratic shocks. We assume that $\bar{A}_t$ evolves as $\bar{A}_t = \bar{A}exp(a_t)$, where $a_t = \rho_a a_{t-1} + \epsilon_{a,t}$, with $\epsilon_{a,t}$ being an i.i.d. shock, while σ_t evolves as $log(\sigma_t/\sigma) = \rho_\sigma log(\sigma_{t-1}/\sigma) + \epsilon_{\sigma,t}$ with $\epsilon_{\sigma,t}$ being an i.i.d. shock. We set ρ_a to 0.9, the standard deviation of $\epsilon_{a,t}$ to 0.0034, ρ_σ to 0.97 and the standard deviation of $\epsilon_{\sigma,t}$ to 0.065. Finally, we assume that the investment installation cost function is quadratic, $(\gamma_I/2)(I_t/I_{t-1} - 1)^2$, where the curvature parameter, γ_I, is set to 20. This last batch of parameters has been calibrated to match some second moments for the main macroeconomic aggregates observed for the US economy in 1985Q1-2019Q4, using a minimum-distance routine based on simulated moments. The model solved under a carbon tax policy can reproduce the observed standard deviation of the GDP, the relatively lower consumption volatility, and the relatively higher volatility of investments observed in the US data over the period 1985Q1-2019Q4. See Appendix C.

4 Business Cycle Fluctuations: Cap-and-trade Versus Tax

In this section, we look at how a cap-and-trade scheme and a carbon tax perform in the face of business cycle uncertainty. We examine the impulse response to isolated shocks

[14] See OECD (2022), Environmental tax (indicator). doi: 10.1787/5a287eac-en (Accessed on 18 May 2022).

Table 1: Calibrated Parameters and Steady-State Ratios

	Description	Value
Steady state ratios and values		
C/Y^n	Private consumption	0.80
I/Y^n	Total investment	0.20
Tr/Y^n	Environmental tax revenues	0.007
H	Hours	0.17
L	Leverage Ratio	2.01
$Z-(1+R)$	Spread p.p.	0.52
$F(\bar{\omega})$	Percent of bankrupt business p/quarter	1.5
M	Stock of concentration of carbon	891
$E/(E+E^{*})$	Share of US emissions	0.20
Standard Macroeconomic Parameters		
β	Discount factor	0.99
δ	Depreciation rate of capital	0.025
α	Capital share	0.4
γ_I	Investment installation cost curvature	20
σ_L	Preference parameter (implied)	0.21
η	Preference parameter (implied)	5.72
RRA	Coefficient of relative risk aversion	2
$\bar{A}$	Total factor productivity (implied)	1.26
Financial Parameters		
μ	Monitoring cost	0.21
$1-\gamma$	Fraction of net worth to households	0.035
σ	Standard deviation of log ω	0.30
Environmental Parameters		
$\bar{M}$	Pre-industrial concentration of carbon	581
δ_M	Decay rate of greenhouse gases	0.0021
χ	Emission intensity parameter (implied)	0.017
ξ	Damage function parameter (implied)	7.86e-06
θ_1	Abatement cost function parameter	1
θ_2	Abatement cost function parameter	2.6
Shocks		
ρ_A	Autocorrelation TFP shock	0.90
ρ_σ	Autocorrelation risk shock	0.97
$sd\ \epsilon_A$	Standard deviation TFP shock	0.0034
$sd\ \epsilon_\sigma$	Standard deviation risk shock	0.065

first, and then the welfare costs of business cycles. In the last part of this section, we abandon the assumption that policymakers must choose between a constant price and a quantity instrument and investigate what happens when we derive pricing and cap rules that allow environmental policy to be set as a function of current economic conditions.

4.1 Dynamic Analysis

We start the dynamic analysis by exploring the response of the economy to a positive one-standard-deviation shock to the TFP under alternative environmental policy regimes. See Figure 1. In response to this expansionary shock, consumption, investment, and net output increase immediately. As the marginal productivity of final-good producers increases, the demand for intermediate goods goes up, pushing firms at the intermediate-good level to expand their production. This leads to the building up of the capital stock, especially after the initial surge in price $Q_{K,t}$ has settled. Due to the initial jump in the return on production, intermediate-good producers have a higher net worth, so their balance sheet shifts more heavily toward capital. Since firms can finance their production activity in a greater part through their own acquired resources, leverage declines.[15] The probability of default falls as the cut-off value $\bar{\omega}_t$ goes down, with monitoring costs following suit, while banks end up reducing the interest rate charged on loans, leading to a decline in the spread. At the basis of this excessive reaction to shocks there is a dynamic pecuniary externality. This occurs because polluting firms do not fully consider the impact of their borrowing behavior on the cost of capital, which in turn affects their ability to borrow. When a large fraction of producers borrow to purchase capital goods, the price of capital increases. As net worth is positively related to the price of capital, the borrowing capacity of producers increases. However, as this externality is not fully accounted for by individual producers, it ultimately results in excessive borrowing and inefficient fluctuations in output.

In addition to the above considerations, the most evident result in Figure 1 is that a cap-and-trade scheme keeps the system substantially more stabilized than a carbon tax policy in the face of an economic upturn. In fact, when there is a carbon tax regime, the overall amount of emissions can respond freely and pro-cyclically to the

[15] Net worth is directly exposed to the increase in the value of capital Q associated with the positive productivity shock, this is because, in our model, net worth refers to the difference between the market value of assets and liabilities of a firm's balance sheet. This explains why leverage moves countercyclically as it also does in other variants of the financial accelerator model (i.e., Gertler and Karadi 2011). However, if capital were valued at its book value, leverage would become procyclical. In this respect, Adrian and Shin (2010) find that book leverage is pro-cyclical for investment bankers, while He et al. (2017) show that market leverage is countercyclical, as in this model.

shock. All the positive effects described above are magnified as the marginal cost related to abatement and the price of emissions remain constant. In contrast, relative compliance costs decrease slightly on impact as a result of the upward jump in the price of capital. In this way, the tax instrument imposes a slightly lower burden on polluters than the quantity instrument. This mechanism gives intermediate goods producers a higher return on their goods, making the cut-off value drop more than it would in the cap-and-trade scenario. Put differently, implementing a carbon tax allows more firms to have enough resources to repay their loans, reducing the probability of bankruptcy. Due to this, banks end up charging lower rates, which helps reduce the spread and make it easier to get credit. That is why we observe that the amount of credit, at a certain point, goes again above its pre-shock level. Furthermore, the expansion of production in the intermediate goods sector is reflected in the marginal cost r^x paid by the final good producers. After an initial increase in this cost, in fact, capital-intensive production inputs become less costly and final output increases further.

Under a cap policy, higher environmental compliance costs limit the increase in the price of capital Q, leading to an attenuated effect on net worth and financial premium. Putting it differently, the dynamic pecuniary externality is weaker. The cost of borrowing declines by much less, investments are less stimulated, and the impact on the price of capital is further reduced. Due to these effects, firms cannot take full advantage of the economic upturn and have less room to repay their loans. This leads to a smaller decrease in the probability of going bankrupt than under a carbon tax policy. As a result, production in the capital-intensive sector increases by less and the marginal cost r^x remains above its steady state level throughout the adjustment toward the steady state. It is interesting to see how these effects pile up during the adjustment process, pushing the risk premium temporarily above its pre-shock level, while the amount of credit shrinks and stays below its pre-shock level, contrary to what we observe under a tax. The financial accelerator mechanism is then somehow reversed under a cap.

We now turn our attention to the dynamic response of the economy to the risk shock. Specifically, Figure 2 shows the negative consequences for the economy of a one-standard-deviation shock to the volatility of idiosyncratic productivity shocks. Higher uncertainty increases the probability of bankruptcy by expanding the size of the left-tail default events. An increase in the probability of a low ω, in turn, pushes banks to raise the interest rate charged on loans to producers, and credit conditions tighten. It follows that intermediate-good producers are bound to purchase less capital. This, in turn, entails lower investments, leading to a contraction in economic activity and consumption. On impact, leverage increases as the decline in the value of the net worth of intermediate firms offsets the drop in the value of their assets.

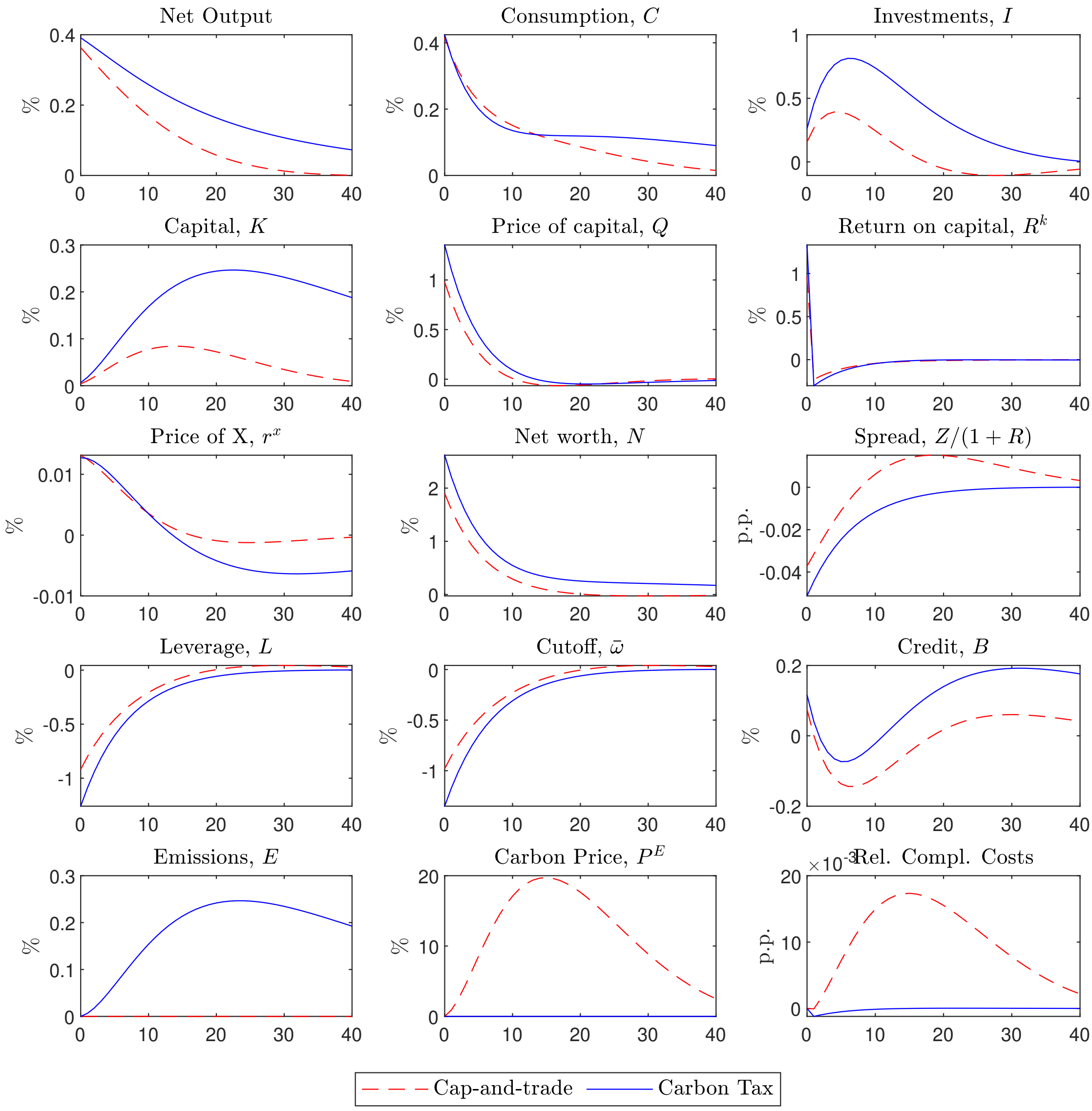
When comparing the behavior of the economy under the two environmental regimes, Figure 2 confirms the above findings, with the economy being much more stable under a carbon trading scheme. In the face of a downturn, a cap-and-trade policy prevents the economy from experiencing a more profound crisis. We observe that under a cap regime, the abatement effort and the price of emissions move pro-cyclically. This allows polluting firms to cut their environmental compliance costs when faced with a crisis. Contrary to what happens in the case of a fixed-price instrument, producers can limit the decline in the return on their production so that the rise of the threshold level of productivity necessary to break even is partially contained. As a consequence, production in the capital-intensive sector reduces by less, and the price paid by final good producers for intermediate inputs increases by less. The propagation channel of this shock from one sector to another is then mitigated. In this sense, a cap policy acts as an automatic stabilizer. At the same time, implementing a carbon tax makes the financial system less stable in the event of an economic slowdown. The asset price Q declines more, and so the amount of credit, while the risk premium stays persistently higher than its pre-shock value. As before, in the backstage, the relative compliance costs sharply decline under a cap, while slightly increase under a tax. Consistently with what is observed in response to a TFP shock, the acceleration mechanism is reversed under a cap during the adjustment path, with the spread, which after an initial increase, now falls below its pre-shock level.

As the discussions have already made clear, the differences between the two alternative regimes are accentuated by the presence of an imperfect financial sector whose mechanisms magnify the response of the economy to shocks under a carbon tax. On the other hand, a cap significantly dampens the financial accelerator and reverses the acceleration mechanism, working in the opposite direction. The following section will shed more light on the subject by shifting the scope of the analysis to the welfare costs of the business cycle.

4.2 Welfare Costs

In this section, we compare the welfare costs of the business cycle of the two environmental policy regimes. The welfare costs of business cycles associated with each environmental policy are calculated as the difference between the welfare under the deterministic steady state and the mean welfare associated with the policy in question. More precisely, we compute the fraction of deterministic steady-state consumption that

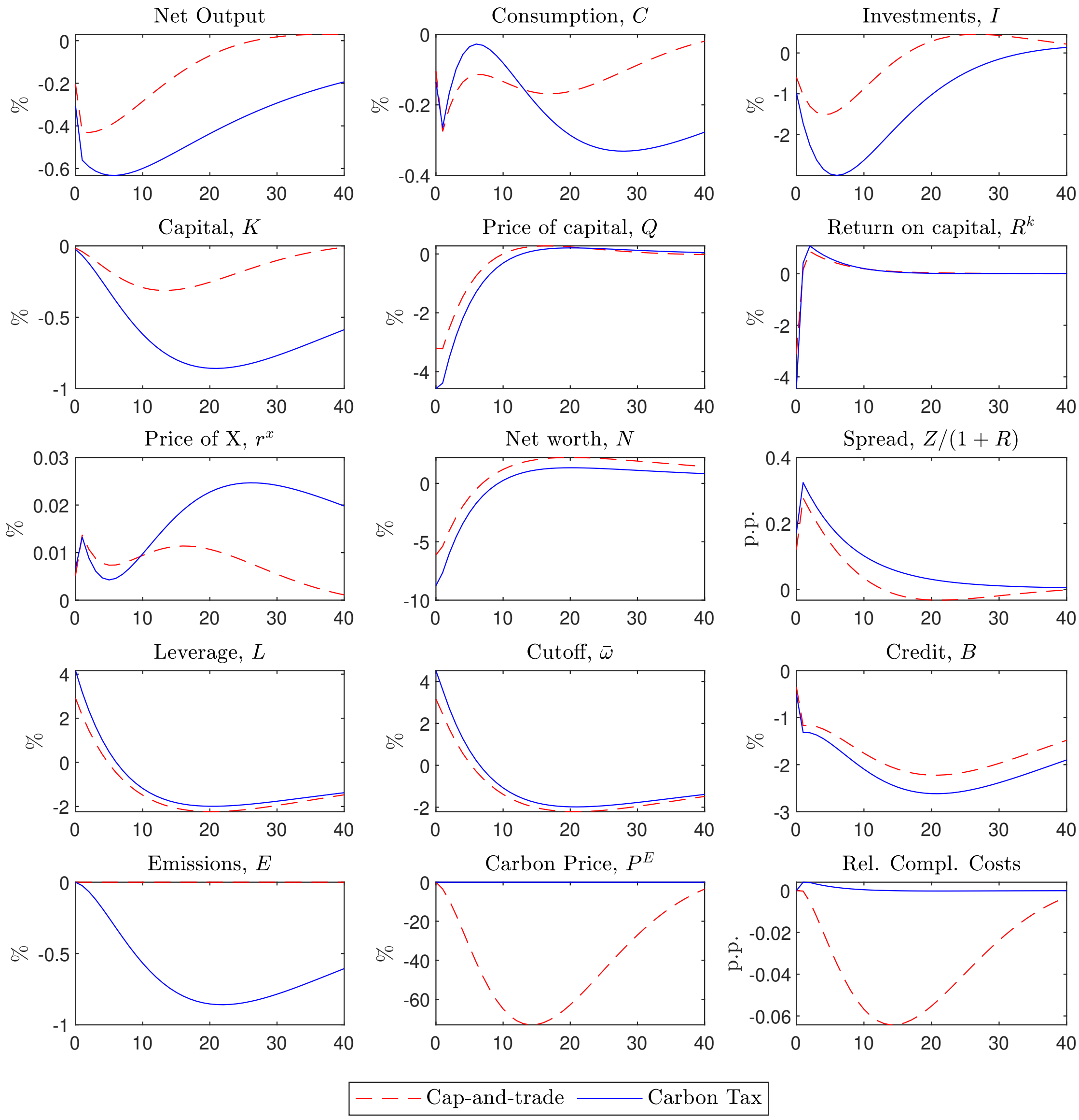
Figure 1: Dynamic Response to a One-Standard-Deviation TFP Shock



Note: Results are reported as percentage deviations from the initial steady state, except for the spread and the relative compliance costs, which are reported as percentage point deviations from the steady state.

households would be ready to give up in order to be indifferent between the corresponding sequences of consumption and hours in the absence of fluctuations and the equilibrium stochastic processes for these two variables associated with the environmental policy under consideration.

Figure 2: Dynamic Response to a One-Standard-Deviation Risk Shock



Note: Results are reported as percentage deviations from the initial steady state, except for the spread and the relative compliance costs, which are reported as percentage point deviations from the steady state.

To this end, we define the welfare associated with the time-invariant equilibrium as $U = u(C, H)/(1 - \beta)$, with $u(C, H)$ being the period-by-period utility specified in 1 and the welfare associated with a particular environmental policy, say EP, as U^{EP}. As a welfare measure, we use the unconditional expectation of lifetime utility, which is:

$$U^{EP} = \mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t u(C_t^{EP}, H_t^{EP})\right\}, \tag{26}$$

where C_t^{EP} and H_t^{EP} are the equilibrium stochastic processes of consumption and hours under a particular environmental policy regime. Thus, the cost of business cycles under a specific policy is given by ς, so that $u(C(1-\varsigma), H)/(1-\beta) = U^{EP}$. It follows that the higher ς, the higher the welfare costs of the business cycle under a particular environmental policy.

Our findings show a significant difference between the two regimes, with the carbon tax resulting in costs that are roughly three times higher than those observed under a cap-and-trade system. Table 2 provides some insight into this result, reporting mean and volatility values for a selection of macroeconomic variables under both environmental regimes, along with the implied welfare costs of the business cycle.[16]

For all macroeconomic variables considered, the price instrument implies higher volatility than the quantity instrument, as expected. As shown by the higher mean and volatility of both the bankruptcy and spread indicators, as well as the lower level of capitalization seen under price regulation, the carbon tax causes more financial instability than the cap. When it comes to the environmental side of the economy, the instruments provide contrasting indications: emissions are lower in the mean under the tax, but they are not stable over the business cycle. At the same time, relative compliance costs are reduced on average under the cap, but they are highly volatile since, as we have seen, they are strongly procyclical. The high volatility of the carbon price is primarily responsible for this effect.[17] Taking into account all these dynamics, our measure of the welfare costs of business cycles driven by TFP and risk shocks appears to favor a cap-and-trade regime.[18]

[16] In Appendix D, we report the results using welfare cost measures based on conditional welfare, and on unconditional and conditional compensating variations of welfare. There, we also provide evidence that results under the benchmark scenario where no policy is in place do not significantly differ from the carbon tax case, so we avoid referring to the former in the subsequent analyses. The fact that the economy behaves in a similar way in the carbon tax and in the no-policy case has already been shown by previous contributions in this area (see, e.g., Annicchiarico and Di Dio, 2015).

[17] The excessive volatility of carbon price is related to the fact that aggregate emissions are kept fixed.

[18] Given the calibrated values assigned to the standard deviations of the shocks, the bulk of fluctuations is ascribed to risk shocks, that in fact account for 95.26% of the welfare costs under a cap and for 96.84% under a tax. It can be shown that if we completely eliminate the risk shock, the differences in terms of welfare will still be significant, but the gap will reduce. Specifically, welfare costs are about 67% higher in the case of taxes compared to a cap. The TFP shock is more 'neutral' because it affects all firms in the same way, and the differences between the two environmental regimes are amplified by the financial acceleration mechanism. On the other hand, the risk shock by changing the size of the left-tail

Table 2: Mean (and Volatility) for a Selection of Variables and Welfare Costs

	Cap-and-Trade	Carbon Tax
Net Output	-0.5691	-2.0569
	(0.0189)	(0.0360)
Consumption	-0.4837	-1.5935
	(0.0115)	(0.0196)
Investment	-0.9034	-3.8697
	(0.0113)	(0.0236)
Bankruptcy	0.5357	0.7028
	(0.0267)	(0.0345)
Net worth	3.0069	0.9986
	(0.7103)	(0.8225)
Spread	0.1573	0.2728
	(0.0082)	(0.0111)
Emissions	-	-4.1252
	-	(0.0073)
Carbon Price	-60.1306	-
	(0.1781)	-
Rel. Compliance Costs	-0.0550	0.0008
	(0.0029)	(0.0001)
Welfare costs	0.6178	1.5231

Note: Mean results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate, the spread and the relative compliance costs, which are reported as percentage point deviations. Standard deviations are in parentheses.

Table 3 reports the welfare costs under the two regimes for different values of a selection of parameters. An increase in volatility σ of the random variable ω brings about a reduction in welfare costs under both scenarios, especially in the case of the carbon tax. In general terms, as the probability of a low ω rises, banks increase the interest rate charged on loans to producers to cover the higher resulting costs, which, in turn, leads to lower borrowing by firms. This effect becomes even more evident in Figure 3: as risk rises, leverage decreases, and the welfare costs of the business cycle converge under the two alternative environmental policies. This exercise sheds light on one of the most important parts of the analysis: when the volatility of production outcomes increases, firms opt to reduce their borrowing, i.e., the channel through which financial accelerator

default events makes the bankruptcy rate of polluting firms more sensitive to environmental policy. The differences are then further amplified by the financial acceleration mechanism.

Table 3: Welfare Costs of the Business Cycle for Different Values of Parameters

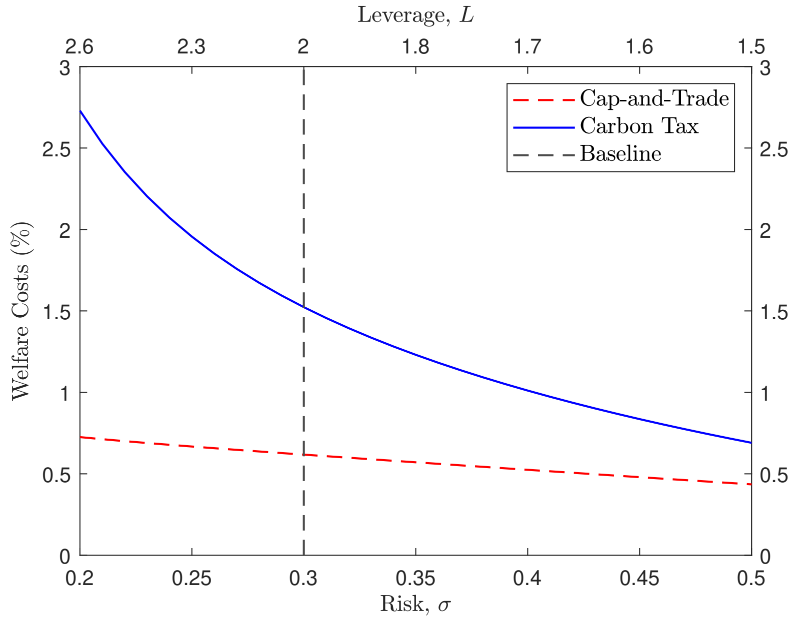
	Cap-and-trade	Carbon Tax
Baseline	0.6178	1.5231
$\sigma = 0.2$	0.7253	2.7296
$\sigma = 0.4$	0.5246	1.0111
$\theta_2 = 2$	0.2477	1.5230
$\theta_2 = 3$	0.7969	1.5233
$\mu= 0.1$	0.5660	1.1905
$\mu = 0.3$	0.6327	1.6679
$\mu = 0.8$	0.6529	2.0039
RRA = 1.5	0.6045	1.4231
RRA = 3	0.6583	1.7854
RRA = 5	0.8618	3.1093
$\gamma_I= 5$	0.5566	1.2131
$\gamma_I = 15$	0.6078	1.4501
$\gamma_I = 25$	0.6247	1.5647

effects propagate. When the financial transmission mechanism weakens, welfare costs unambiguously decrease and do so more intensively under the scenario in which financial effects are amplified.

Changing the coefficient of the abatement cost function by increasing θ_2 leaves the welfare costs under a carbon tax substantially unchanged, as the marginal abatement effort remains constant throughout the business cycle due to the fixed price for emissions. On the other hand, uncertainty, according to Jensen's inequality, implies higher average marginal costs for polluting firms in a cap-and-trade system. For this reason, the stabilizing effect of the cap is reduced and the policy costs more in terms of welfare than in the benchmark case. This makes it more like a carbon tax.

Higher monitoring costs μ intensify the imperfections of financial markets, since banks lose efficiency in collecting revenues and, consequently, lending. As the cost of monitoring goes up, the return on production for polluting firms moves further away from the risk-free interest rate, leverage goes down and the financial market failure becomes deeper. In particular, when monitoring costs are high, the premium on external funds becomes more sensitive to changes in the leverage position. In dynamic terms, increased financial market imperfections eventually induce greater economic volatility. As a result, welfare costs rise as μ increases, and even more so under the less stable scenario of a

Figure 3: Welfare Costs of the Business Cycle over Different Values of Risk and Leverage



Note: The top axis, reporting the leverage ratio, has been re-scaled for illustrative purposes.

carbon tax. The effects are milder under a cap-and-trade regime because, as explained in the previous section, under a quantity restriction on emissions, the financial acceleration mechanism is partially reversed.[19]

As expected, a higher coefficient of relative risk aversion entails higher welfare costs, especially with tax regulation. As households become less willing to take risks, their desire for smooth consumption goes up, and so do the costs of welfare. They do so more in a regime that is more exposed to risk and fluctuations throughout the business cycle.

Finally, Table 3 indicates that a greater curvature in the investment installation function results in increased welfare costs of fluctuations and greater discrepancies between the two environmental regulations. Specifically, since capital is the sole source of net worth for polluting firms, a more pronounced curvature in the technology used to convert final goods into capital causes higher pecuniary externalities that intensify fluctuations in net worth and credit in response to shocks, especially under a carbon tax.

Our simulation results point to substantial differences between the two environmental

[19] We do not discuss here what could bring about an increase in μ. Higher costs can potentially reflect stricter monitoring activity by banks conditional on economic conditions or policies imposed by the government. In this sense, μ could even be treated as endogenous to the problem, but we leave a more structured analysis of this kind for future research.

regimes. The fact that permit prices react endogenously to the business cycle favors cap-and-trade programs over taxes. This is why, in the next section, we focus on optimal quantity and price rules for environmental policy, whose level of stringency changes with the business cycle.

4.3 Optimal Environmental Policy Rules

So far, we have looked at environmental policies that, like most real-world policies, do not explicitly respond to economic fluctuations and maintain a constant level of stringency across cycles.[20] In this section, we derive simple hybrid environmental policy rules according to which the cap on emissions or the carbon tax is allowed to vary in response to economic fluctuations. Since emission abatement is costly and damages from pollution are roughly smooth over the business cycle, it makes sense to design a flexible environmental policy rule that responds to short-run market conditions. To this end, we start by considering a hybrid environmental regulation according to which the level of the cap adjusts endogenously to the deviation of the net output from its non-stochastic steady state, that is:

$$E_t = \bar{E}\left(\frac{Y_t^n}{Y^n}\right)^{\nu}, \tag{27}$$

where variables without subscripts are steady-state values. To discipline the way we parameterize these rules, we adopt a welfare criterion and search for the value of the parameter ν that minimizes welfare costs. In this sense, these hybrid rules are optimal. [21] The first panel of Figure 4 displays the result and shows that the relationship between the policy parameter ν and the welfare cost is convex, with a minimum cost for ν at -2.3380. This result would suggest that a certain degree of temporal flexibility of the cap is desirable, with the cap level moving countercyclically in an attempt to mitigate the negative impact of uncertainty on bank net worth and reduce the bankruptcy rate. The welfare costs of the business cycle decline significantly under this optimal cap policy, as shown in Table 4, however the price of permits becomes more volatile, and on average is higher than in the fixed cap case.

Using the same method, we then look for the optimal carbon tax rule, where the tax rate is set up to automatically adjust to changes in economic conditions:

[20] Business-cycle smoothing can be considered of second-order importance, in contrast to the need to set the right degree of stringency of regulation that is instead seen as of first-order importance to limit environmental damages. See Annicchiarico et al. (2022a) for a discussion.

[21] In our grid search for the optimal policy parameters ν is restricted to lie in the interval [-60, 60], with a step of size 0.0005.

$$P_t^E = \bar{P}^E \left(\frac{Y_t^n}{Y^n}\right)^{\tau}, \tag{28}$$

where τ is the policy parameter that governs the response of the carbon tax. Figure 4 shows that the welfare costs of business cycles are minimized at $\tau = 52.2245$. Consistently with the previous result, the hybrid carbon tax then prescribes a vigorous response to current economic conditions with a significant reduction in welfare costs.[22] Also, under this rule, the price of carbon is allowed to decrease when net output goes down, that is to say, environmental regulation becomes permissive during recessions and strict during expansions. Because of this lean-against-the-wind policy, the welfare cost of business cycles is much lower, and the distorting effects of macroeconomic volatility are less severe than they would be if carbon prices were kept the same.

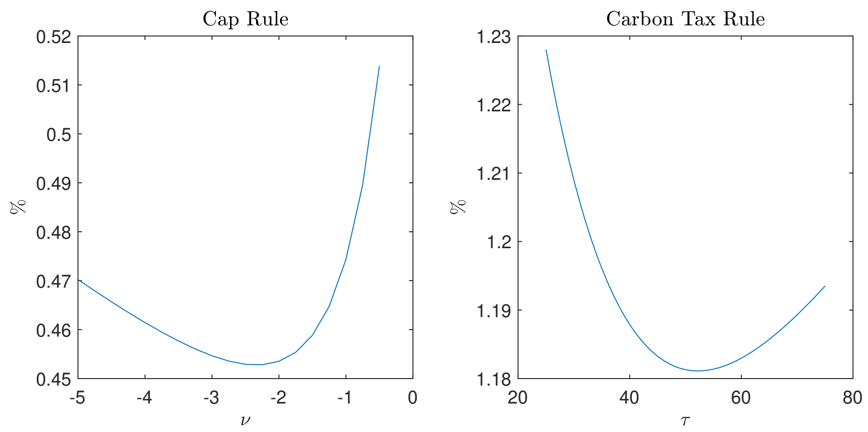
In both cases, environmental regulation is designed to reduce the effects of the financial accelerator, stabilizing the economy in the face of shocks. However, this course of action may result in increased fluctuations in the cost of carbon under the quantity regulation. In addition, as we can see from Table 4, welfare costs of the business cycle are still high and significantly different between the two carbon pricing rules.[23] It is essential to keep in mind that this is an indirect way to address financial frictions and reduce the amplitude of macroeconomic fluctuations. The most common approach to dealing with financial instability is to use policy tools that are specifically designed for the task.[24] This is why in the next section, we modify the baseline model by introducing macroprudential policy and explore whether there is scope to align the business cycle properties of environmental policies.

[22] This strong reactivity of the carbon price to net output suggests that the optimal tax rule is designed to mimic the behavior of a cap, where the reactivity of the permit price to economic fluctuations is similar in order of magnitude. See Figures 1 and 2. As in the cap rule, in our grid search for the optimal policy parameters τ is restricted to lie in the interval [-60, 60], with a step of size 0.0005.

[23] In Appendix E, we show the dynamic response of the economy to shocks under optimal environmental policy rules.

[24] Unlike what is presented in this section, the standard design of climate policies should mainly concern itself with the achievement of climate targets. Reducing the financial frictions that amplify shocks and induce high volatility is not among the goals of climate policy. However, in exceptional circumstances, countercyclical climate policies have been considered in both policy and academic circles as a way to cushion economic shocks, with the recent rise in energy prices serving as an example. For a discussion of countercyclical environmental regulations, see Dominioni and Faure (2022). For a theoretical analysis of the optimality of a dynamic regulation of a carbon emissions market system, see, e.g., Aïd and Biagini (2023).

Figure 4: Welfare Costs of the Business Cycle Under Variable Environmental Policy Rules



5 Macroprudential Regulations

In the last section, we talked about how important it is to look at the financial channel when comparing welfare costs under cap-and-trade and carbon tax policies. It is now worth considering whether the implementation of a financial regulatory system can play a significant role in aligning the performance of various carbon pricing schemes, reducing the uncertainty surrounding their operation throughout the business cycle.

5.1 Reserve Requirements

Following Leduc and Natal (2018), we introduce a macroprudential policy similar to reserve requirements for lending institutions. In particular, we assume that banks are required to keep a portion of their funds in reserves, which are assumed to be in "cash" and earn a zero rate of return.[25] Analytically, this leads to the following rewriting of the cash constraint for banks:

$$
\begin{aligned}
&(1 - F_t(\bar{\omega}^N_{t+1}))B^N_{t+1}Z^N_{t+1} + (1-\mu)\int_0^{\bar{\omega}^N_{t+1}} \omega dF_t(\omega)(1+R^k_{t+1})Q_{K,t}K^N_{t+1} \\
&= \frac{B^N_{t+1}}{\Phi_t}\left(1+R_t\right),
\end{aligned}
\tag{29}
$$

[25] Notably, in most countries, banks are required to hold a fraction of their funds as liquid assets on accounts at their national central bank. These are known as "minimum reserves" and function as a valve, allowing banks to face short-term liquidity needs and unexpected changes in the interbank market, where banks lend to each other.

Table 4: Means (and Volatility) and Welfare Costs under Optimal Environmental Policy Rules

	Cap-and-Trade	Carbon Tax
Net Output	-0.5325 (0.0108)	-1.5911 (0.0269)
Consumption	-0.4171 (0.0082)	-1.2266 (0.0149)
Investment	-0.9843 (0.0056)	-3.0171 (0.0176)
Bankruptcy	0.3113 (0.0182)	0.5975 (0.0294)
Net worth	1.0564 (0.6622)	0.9824 (0.7166)
Spread	0.1135 (0.0075)	0.2245 (0.0090)
Emissions	1.2752 (0.0034)	-3.0437 (0.0044)
Carbon Price	-3.6366 (0.3821)	-5.3199 (0.0788)
Rel. Compliance Costs	-0.0151 (0.0062)	-0.0090 (0.0012)
Welfare costs	0.4528	1.1811

Note: Mean results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate, the spread and the relative compliance costs, which are reported as percentage point deviations. Standard deviations are in parentheses.

where Φ_t defines the fraction of deposits that banks can loan out. In other words, financial intermediaries must now issue B_{t+1}/Φ_t deposits to finance a quantity B_{t+1} of loans to firms.[26]

Reserve requirements are set up so that, as financial activity accelerates, conditions to extend loans tighten. This leads to the following general rule:

$$\Phi_t = \Phi^\star \left(FI_t\right)^{-\psi}, \quad \Phi_t \in (0,1], \tag{30}$$

where FI_t is an indicator of the level of financial activity. In the absence of the reserve requirement, $\Phi_t = 1$, we retrieve the case analyzed until now. We set $\Phi^\star$ to 0.98, so that

[26] In equilibrium, now the condition $B_{t+1} = \Phi_t B_t^H$ must be satisfied.

Table 5: Welfare Costs of the Business Cycle under Macroprudential Regulations - Reserve Requirements

Cap-and-Trade		
	Reserve Requirement	Policy Mix
Baseline $\Phi_t = 1$	0.6178	0.4528 $\nu = -2.3380$
$\Phi_t = \Phi^\star$	0.1957	0.1883 $\nu = -0.3695$
$\Phi_t = \Phi^\star \left(\frac{B_{t+1}}{B_t}\right)^{-\psi^B}$	0.1207 $\psi^B = 1.0465$	0.1164 $\psi^B = 1.0482$, $\nu = -0.3011$
$\Phi_t = \Phi^\star \left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-\psi^Q}$	0.1807 $\psi^Q = 0.7220$	0.1776 $\psi^Q = 0.6890$, $\nu = -0.2391$
Carbon Tax		
	Reserve Requirement	Policy Mix
Baseline $\Phi_t = 1$	1.5231	1.1811 $\tau = 52.2245$
$\Phi_t = \Phi^\star$	0.3863	0.3455 $\tau = 24.2990$
$\Phi_t = \Phi^\star \left(\frac{B_{t+1}}{B_t}\right)^{-\psi^B}$	0.3231 $\psi^B = 0.9935$	0.2695 $\psi^B = 1.0475, \tau = 44.6699$
$\Phi_t = \Phi^\star \left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-\psi^Q}$	0.2300 $\psi^Q = 0.6790$	0.2168 $\psi^Q = 0.7000, \tau = 9.1723$

when $\psi = 0$, a static rule is applied that implies a reserve requirement of 2%.[27]

As Table 5 shows, introducing a similar static regulation already goes a long way in reducing welfare costs under both environmental policies and making them converge. As banks are limited in the amount of funds they can convert into loans, credit is reduced in the economy, and firms have fewer resources to invest, which leads to lower leverage. As a result, the effect of the financial accelerator is mitigated, and the fluctuations of the business cycle are strongly dampened. Now, we explore the role of dynamic macroprudential regulation, where the parameter defining the sensitivity to financial activity indicators is optimally set to minimize welfare costs of business cycles, while the steady-state level of reserves is kept at 2%. Note that here we are not interested in computing the welfare maximizing policy in absolute terms, but rather the one that minimizes the welfare cost of fluctuations around a distorted steady state. As a first dynamic macro-

[27] As in Leduc and Natal (2018), we opt to set the reserve requirement at 2% to reflect the average rate observed in industrialized countries in the period 1975-2011, as documented by Federico et al. (2014). Since then, this ratio has been significantly reduced by several central banks (i.e., to 1% in the euro area and 0% in the US).

prudential regulation, reserve requirements are allowed to vary countercyclically with respect to credit growth:

$$\Phi_t = \Phi^\star \left(\frac{B_{t+1}}{B_t}\right)^{-\psi^B}. \tag{31}$$

Credit growth represents an immediate choice as a financial indicator, given its role in amplifying economic imbalances in periods leading to financial crises (see, i.e., Schularick and Taylor 2012).[28] In this way and for our purposes, tying the reserve requirement to credit growth is a way to introduce macroprudential regulation that fits with this model setting and the spirit of the Basel III accord, which calls for countercyclical capital requirements when credit growth is high.[29]

As a second option, we also link reserve requirements with the variation in asset prices:

$$\Phi_t = \Phi^\star \left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-\psi^Q}. \tag{32}$$

As the price of assets in the model reflects the conditions at which capital is traded and, consequently, net worth levels, it represents another straightforward indicator summarizing the movements in the overall financial activity.[30]
Table 5 shows that implementing either of the two optimal dynamic rules improves static regulation: welfare costs decrease significantly under both environmental scenarios. This is because financial regulation is designed to engineer a procyclical response in the spread that keeps the economy stable and reverses the financial accelerator mechanism.[31] Under a cap-and-trade system, it is interesting to note that an optimal rule that reacts to credit growth has lower welfare costs than a rule that reacts to asset price growth. Under a tax policy, the reverse is true. The amount of credit is a predetermined variable that moves more smoothly in response to shocks, while the asset price is a forward-looking variable that is more reactive to shocks. Under a carbon tax, where the economy is more

[28] We also explore the option of credit-to-GDP gaps, as they have been shown to be reliable indicators to calculate capital buffers through a mechanical macroprudential policy rule, as shown in Alessandri et al. (2022). Since the results do not differ significantly, we only include the credit growth rule in our discussion.

[29] The use of reserve requirements as a macroeconomic stabilization tool is well documented in Federico et al. (2014) who show that mainly emerging economies have used this tool. However, in the face of the recent coronavirus crisis, the Federal Reserve intervened with a wide range of actions to keep credit flowing, including the elimination of bank reserve requirements.

[30] In our search for the optimal macroprudential rule, the policy parameters ψ^B and ψ^Q are restricted to lie in the interval [-5, 5], with a step of size 0.0005. Since $\Phi_t \in (0, 1]$, we rule out solutions such that $\Phi_t + 2\sigma_\Phi > 1$, where σ_Φ is the standard deviation of Φ_t.

[31] In Appendix F, we show the response of the economy to TFP and risk shocks under the optimal credit rule for both environmental regimes.

unstable and requires more vigorous stabilizing macroprudential regulation, the welfare costs are more effectively minimized under a reserve requirement rule that is anchored to a more volatile variable. In contrast, the opposite is true for the cap.

In the last column of Table 5, we derive the optimal policy mix, where the sensitivity parameters of macroprudential and environmental rules are jointly set to minimize the welfare cost of business cycles. As expected, when both policy rules are optimal, welfare costs are lower, but the results are not significantly different from those obtained when only macroprudential rules are optimized.[32]

Finally, to explore whether macroprudential policy can alleviate the uncertainty inherent to the chosen environmental policy tool, in Table 6 we report the coefficient of variation for emissions, CV_E, and the permit price, CV_{PE}, under different combinations of policies. The results show the non-trivial role of macroprudential regulation in fostering symmetry between quantity and price in environmental regulations. When reserve requirements are added to a cap-and-trade system, the prices of allowances become much less volatile. Similarly, under a carbon tax, financial regulation substantially stabilizes emissions, making the use of price regulation less uncertain. With dynamic financial rules, volatility declines even more, especially when the reserve requirement is adjusted to credit growth.

When both policies are set to minimize the welfare cost of fluctuations, under a cap, we see slightly higher volatility in permit prices than when only the macroprudential policy is optimally set. This is because, as previously discussed in Section 4, a flexible cap increases permit price volatility, although to a very limited extent, in an attempt to smooth out fluctuations in the other macroeconomic variables. On the other hand, under a tax, moving from optimal financial rules to jointly optimal policy rules reduces emissions volatility even further. Again, the regulator is concerned with macroeconomic aggregate stability, even if it means having moderately variable emission allowance prices.

To conclude this section, it is important to stress that in this model, by introducing reserve requirements, we are exacerbating the under-borrowing problem of the economy. Putting it differently, due to macroprudential policy, the economy is more stabilized but around a more distorted steady state.[33] This is why in the next section we briefly show what happens to the relative performance of environmental policy rules when we

[32] In Appendix F, for robustness, we consider other policy mix experiments, where environmental and macroprudential rules are optimally set conditionally on each other. In all cases, the main stabilizing role is played by financial regulation.

[33] The distortion can be quantified in terms of the welfare loss associated with the static reserve requirement; this is equal to 7.48% of the consumption path of the economy without macroprudential policy.

Table 6: Volatility of Emissions and Permit Prices under Macroprudential Regulations - Reserve Requirements

Cap-and-Trade	Reserve Requirement	Policy Mix
Baseline $\Phi_t = 1$	$CV_E = 0$ $CV_{PE} = 8.3578$	$CV_E = 0.0255$ $CV_{PE} = 7.4192$
$\Phi_t = \Phi^\star$	$CV_E = 0$ $CV_{PE} = 2.5125$	$CV_E = 0.0036$ $CV_{PE} = 2.7692$
$\Phi_t = \Phi^\star \left(\frac{B_{t+1}}{B_t}\right)^{-\psi^B}$	$CV_E = 0$ $CV_{PE} = 1.0935$	$CV_E = 0.0025$ $CV_{PE} = 1.3116$
$\Phi_t = \Phi^\star \left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-\psi^Q}$	$CV_E = 0$ $CV_{PE} = 2.3682$	$CV_E = 0.0024$ $CV_{PE} = 2.5190$
Carbon Tax	**Reserve Requirement**	**Policy Mix**
Baseline $\Phi_t = 1$	$CV_E = 0.0574$ $CV_{PE} = 0$	$CV_E = 0.0343$ $CV_{PE} = 1.5576$
$\Phi_t = \Phi^\star$	$CV_E = 0.0273$ $CV_{PE} = 0$	$CV_E = 0.0220$ $CV_{PE} = 0.3962$
$\Phi_t = \Phi^\star \left(\frac{B_{t+1}}{B_t}\right)^{-\psi^B}$	$CV_E = 0.0153$ $CV_{PE} = 0$	$CV_E = 0.0090$ $CV_{PE} = 0.4956$
$\Phi_t = \Phi^\star \left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-\psi^Q}$	$CV_E = 0.0284$ $CV_{PE} = 0$	$CV_E = 0.0262$ $CV_{PE} = 0.1591$

Note: The table reports the coefficient of variation as a measure of volatility.

replace the reserve requirement regulation with a tax-financed government policy capable of circumventing financial frictions and moving the economy toward a more efficient equilibrium. Note that in the real world these costly policy interventions are rarely available, which is why we will only briefly touch on these alternative macroprudential policy instruments.

5.2 Interest Rate Subsidy to Depositors

As emphasized in Section 2, the economy is characterized by too little borrowing. This is because the risk-free rate R, at which households make their deposit decisions, is equal to the average and not to the higher marginal return on production. As a result, the risk-free rate does not offer households a sufficiently strong incentive to save. In these circumstances, an interest rate subsidy to depositors (or equivalently a subsidy to banks' earnings), say s, can move the economy toward a more efficient steady-state

Table 7: Welfare Costs of the Business Cycle under Macroprudential Regulations - Interest Rate Subsidy to Depositors

Cap-and-Trade		
	Subsidy to Depositors	Policy Mix
Baseline $s_t = 0$	0.6178	0.4528 $\nu = -2.3380$
$s_t = s^{\star}$	1.1028	0.4973 $\nu = -5.9730$
$1 + s_t = (1 + s^{\star}) \left(\frac{B_{t+1}}{B_t}\right)^{-\varkappa}$	0.2506 $\varkappa = 1.3190$	0.1926 $\varkappa = 1.2060$, $\nu = -1.1500$
Carbon Tax		
	Subsidy to Depositors	Policy Mix
Baseline $s_t = 0$	1.5231	1.1811 $\tau = 52.2245$
$s_t = s^{\star}$	3.5597	2.2311 $\tau = 60$
$1 + s_t = (1 + s^{\star}) \left(\frac{B_{t+1}}{B_t}\right)^{-\varkappa}$	0.4706 $\varkappa = 1.3330$	0.4366 $\varkappa = 1.3195$, $\tau = 9.6200$

equilibrium, so that the relevant gross interest rate at which savings decisions are taken is now $(1 + R_t)(1 + s_t)$. See Christiano and Ikeda (2011).

In our calibrated model under a carbon tax, we find that the expected welfare is maximized with a subsidy of 1.00% percent, with a welfare gain in consumption equivalent units of around 4.19%. We then compute the welfare cost of business cycles under both environmental policies, as done in the previous section, using the new steady state as a reference point. The results are reported in Table 7. We find that around a more efficient steady state, because of the higher leverage, the welfare cost of business cycles is higher than in the benchmark case and still significantly different between the two environmental policies.[34] Under an interest rate subsidy rate of 1% the welfare cost of business cycles is around 1.1% under a cap and around 3.6% under a tax. To narrow the gap between the two rules, it is necessary to introduce a dynamic subsidy that responds to macroeconomic conditions in the spirit of what was done in the previous section. A simple rule according to which the interest rate subsidy responds to credit growth is

[34] In Appendix F we compare the steady-stave values for some relevant variables under the two different macroprudential policies.

shown to work well in this direction:

$$1 + s_t = (1 + s^\star) \left(\frac{B_{t+1}}{B_t} \right)^{-\varkappa}. \tag{33}$$

In Table 7 we see that the above state-contingent interest rate subsidy to depositors, whether implemented in isolation or in combination with a dynamic environmental policy, reduces the welfare cost of business cycles under both climate actions. Intuitively, by encouraging saving when credit declines or discouraging it when credit increases, a dynamic subsidy that leans against the wind can stabilize the economy. The high cost of business cycles observed under a carbon tax explains why it would be optimal to have a strongly countercyclical pricing of carbon, with an optimal $\varkappa$ that hits the upper bound of our grid-search. A dynamic subsidy makes, *de facto*, the return to the lender state contingent and cushions households from fluctuations in consumption.

6 Conclusions

Even though economists and policy analysts generally agree that ambitious climate actions are needed to reduce greenhouse gas emissions and limit climate disasters, considerable debate continues on the choice of measures to address this problem. In the presence of uncertainty, cap-and-trade schemes and carbon taxes are unlikely to deliver the same outcome. Therefore, assessing their comparative performance becomes more challenging and yields ambiguous results.

This paper contributes to the vast debate on the choice of policy instruments under uncertainty. We compare the welfare cost of business cycles under quantity and price regulations in an environmental dynamic stochastic general equilibrium model featuring heterogeneous polluting firms that borrow from banks and may default.

We find that financial market distortions make the choice of the climate policy instrument more relevant by amplifying the difference in the performance of the two instruments over the business cycle. Under a carbon tax system, the financial accelerator mechanism makes the economy more prone to short-run fluctuations. In contrast, since under a cap-and-trade this mechanism is reversed, financial frictions tend to stabilize business cycle fluctuations. As a result, a cap-and-trade scheme behaves like an automatic stabilizer and, importantly, entails substantially lower welfare costs of business cycles than a carbon tax. Nevertheless, it is worth noting that the ability of this policy to dampen fluctuations crucially depends on the degree of leverage of the economy: the lower the leverage, the smaller, and the closer the welfare costs of fluctuations are under the two alternative environmental policy regimes.

Based on these findings, the paper explores the role of countercyclical macroprudential policies. We find that a macroprudential rule that ties reserve requirements to credit growth or asset prices forces a smoothed response of real variables under both environmental regimes: it significantly reduces the welfare cost of business cycles and their differences across policy regimes.A time-varying interest rate subsidy aimed at encouraging savings when the level of credit declines is shown to work in the same direction. In other words, macroprudential policy, even without any green-biased component, can create a better welfare environment for the implementation of climate policy. A macroprudential regulatory framework that weakens the strength of the distorting effects of financial frictions can stabilize the economy and can *de facto* align the performance of different carbon pricing schemes, reducing the uncertainty surrounding their functioning throughout the business cycle. These results have important policy implications for the potential role central banks and financial regulators can play during the transition to a low-carbon economy. By stabilizing financial markets and the economy, central banks and financial regulators can help reduce the uncertainty inherent to each climate policy tool, expanding the array of policy options on the table.

Future research could further investigate the impact of financial market distortions on the effectiveness of carbon pricing schemes under different economic and climate conditions, such as higher uncertainty in the price of polluting inputs or significant risks of climate disasters. Another line of research could investigate the potential for hybrid policy approaches, like combining carbon pricing schemes with subsidies or other incentives, to address the challenges posed by financial market distortions and improve the effectiveness of climate policies. Finally, it would be interesting to link firms' default risk to climate change-induced natural disasters, as well as to risks associated with technological advancements and ecological transitions that could potentially lead to assets stranding.

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Appendix A

This appendix gives details on the derivation of the debt contract chosen by intermediate-good producers. First, let us recall some relevant functions:

$$
\begin{aligned}
G_t(\bar{\omega}^N_{t+1}) &= \int_0^{\bar{\omega}^N_{t+1}} \omega dF_t(\omega) \\
\Gamma_t\left(\bar{\omega}^N_{t+1}\right) &= (1 - F_t(\bar{\omega}^N_{t+1}))\bar{\omega}^N_{t+1} + G_t(\bar{\omega}^N_{t+1}), \\
G'_t(\bar{\omega}^N_{t+1}) &= \frac{d}{d\omega}\int_0^{\bar{\omega}^N_{t+1}} \omega dF_t(\omega) = \bar{\omega}^N_{t+1} F'_t(\bar{\omega}^N_{t+1}), \\
\Gamma'_t\left(\bar{\omega}^N_{t+1}\right) &= 1 - F_t(\bar{\omega}^N_{t+1}) - \bar{\omega}^N_{t+1} F'_t(\bar{\omega}^N_{t+1}) + G'_t(\bar{\omega}^N_{t+1}) = 1 - F_t(\bar{\omega}^N_{t+1}).
\end{aligned}
$$

Intermediate good producers value a particular debt contract according to the expected return from operating risky technology over the return from depositing net worth in a bank, that is:

$$
\frac{\mathbb{E}_t\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\left[\omega(1+R^k_{t+1})Q_{K,t}K^N_{t+1} - B^N_{t+1}Z^N_{t+1}\right]dF_t(\omega)\right\}}{N\left(1+R_t\right)}. \tag{A-1}
$$

Recall $\bar{\omega}^N_{t+1}K^N_{t+1}\left[(1+R^k_{t+1})Q_{K,t}\right] = B^N_{t+1}Z^N_{t+1}$, then:

$$
\frac{\mathbb{E}_t\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\left(\omega - \bar{\omega}_{t+1}\right)(1+R^k_{t+1})Q_{K,t}K^N_{t+1}dF_t(\omega)\right\}}{N\left(1+R_t\right)}.
$$

Using $Q_{K,t}K^N_{t+1} = N + B^N_{t+1}$ and $L^N_t = (N + B^N_{t+1})/N$, we get:

$$
\begin{aligned}
&\frac{\mathbb{E}_t\left\{\int_{\bar{\omega^N}_{t+1}}^{\infty}\left(\omega - \bar{\omega}_{t+1}\right)(1+R^k_{t+1})L^N_t N dF_t(\omega)\right\}}{N\left(1+R_t\right)} = \\
&\mathbb{E}_t\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\omega dF_t(\omega) - \int_{\bar{\omega}^N_{t+1}}^{\infty}\bar{\omega}^N_{t+1}dF_t(\omega)\right\}\frac{1+R^k_{t+1}}{1+R_t}L^N_t.
\end{aligned}
$$

Solving the second integral by parts, we obtain:

$$
\begin{aligned}
&\mathbb{E}_t\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\omega dF_t(\omega) - \bar{\omega}^N_{t+1}F(\infty) + \bar{\omega}^N_{t+1}F_t(\bar{\omega}^N_{t+1}) - \int_{\bar{\omega}^N_{t+1}}^{\infty} 0 dF_t(\omega)\right\}\frac{1+R^k_{t+1}}{1+R_t}L^N_t = \\
&\mathbb{E}_t\left\{1 - G_t(\bar{\omega}^N_{t+1}) - (1 - F_t(\bar{\omega}^N_{t+1}))\bar{\omega}^N_{t+1}\right\}\frac{1+R^k_{t+1}}{1+R_t}L^N_t = \\
&\mathbb{E}_t\left[1 - \Gamma_t\left(\bar{\omega}^N_{t+1}\right)\right]\frac{1+R^k_{t+1}}{1+R_t}L^N_t.
\end{aligned} \tag{A-2}
$$

On the side of banks, we start from the cash constraint

$$
\begin{aligned}
&(1 - F(\bar{\omega}^N_{t+1}))B^N_{t+1}Z^N_{t+1} + (1-\mu)\int_0^{\bar{\omega}^N_{t+1}}\omega dF_t(\omega)(1+R^k_{t+1})Q_{K,t}K^N_{t+1} \\
&= B^N_{t+1}\left(1+R_t\right).
\end{aligned} \tag{A-3}
$$

Recalling the cut-off definition $\bar{\omega}^N_{t+1} K^{N_{t+1}}_{t+1} \left[(1+R^k_{t+1})Q_{K,t}\right] = B^{N_{t+1}}_{t+1} Z^N_{t+1}$ again and $L^N_t = Q_{K,t} K^N_{t+1}/N$ or $L^N_t = (N + B^N_{t+1})/N$, we can rewrite (A-3) as:

$$(1 - F_t(\bar{\omega}^N_{t+1}))\bar{\omega}^N_{t+1} + (1-\mu) \int_0^{\bar{\omega}^N_{t+1}} \omega dF_t(\omega) = \frac{1+R_t}{1+R^k_{t+1}} \frac{B^N_{t+1}}{Q_{K,t} K^N_{t+1}} =$$
$$(1 - F_t(\bar{\omega}^N_{t+1}))\bar{\omega}^N_{t+1} + (1-\mu) \int_0^{\bar{\omega}^N_{t+1}} \omega dF_t(\omega) = \frac{1+R_t}{1+R^k_{t+1}} \frac{L^N_t - 1}{L^N_t}.$$

Making use of the previously defined functions, we can eventually recast the zero-profit condition for banks as:

$$\Gamma_t\left(\bar{\omega}^N_{t+1}\right) - \mu G_t(\bar{\omega}^N_{t+1}) = \frac{1+R_t}{1+R^k_{t+1}} \frac{L^N_t - 1}{L^N_t} \tag{A-4}$$

which can also be rewritten in terms of leverage as:

$$L^N_t = \frac{1}{1 - \frac{1+R^k_{t+1}}{1+R_t}\left[\Gamma_t\left(\bar{\omega}^N_{t+1}\right) - \mu G_t(\bar{\omega}^N_{t+1})\right]} \quad \text{or} \tag{A-5}$$
$$Q_{K,t} K^N_{t+1} = \frac{N}{1 - \frac{1+R^k_{t+1}}{1+R_t}\left[\Gamma_t\left(\bar{\omega}^N_{t+1}\right) - \mu G_t(\bar{\omega}^N_{t+1})\right]}.$$

Intermediate-good producers finally choose the contract that maximizes their objective (A-2) among the $(\bar{\omega}^N_{t+1}, L^N_t)$ combinations that satisfy (A-5). Since the constraint is independent of net worth, we get rid of the superscript N from now on, and we have the following:

$$\max_{\{L_t, \bar{\omega}_{t+1}\}} \mathbb{E}_t\left[1 - \Gamma_t\left(\bar{\omega}_{t+1}\right)\right] \frac{1+R^k_{t+1}}{1+R_t} L_t$$
$$\text{s.t.} \quad L_t = \frac{1}{1 - \frac{1+R^k_{t+1}}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]}.$$

The problem can be rephrased as follows:

$$\max_{\bar{\omega}_{t+1}} \frac{1}{1+R_t} \mathbb{E}_t(1+R^k_{t+1}) \frac{\left[1 - \Gamma_t\left(\bar{\omega}_{t+1}\right)\right]}{1 - \frac{1+R^k_{t+1}}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]}.$$

The first-order condition is then found to be:

$$\frac{\left[-\Gamma'_t\left(\bar{\omega}_{t+1}\right)\right]\left(1 - \frac{1+R^k_{t+1}}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]\right)}{\left(1 - \frac{1+R^k_{t+1}}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]\right)^2} +$$
$$- \frac{\left[1 - \Gamma_t\left(\bar{\omega}_{t+1}\right)\right]\left(-\frac{1+R^k_{t+1}}{1+R_t}\left[\Gamma'_t\left(\bar{\omega}_{t+1}\right) - \mu G'_t(\bar{\omega}_{t+1})\right)\right)}{\left(1 - \frac{1+R^k_{t+1}}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]\right)^2} = 0.$$

Rearranging, we get:

$$\frac{\Gamma'_t\left(\bar{\omega}_{t+1}\right)}{1-\Gamma_t\left(\bar{\omega}_{t+1}\right)}=\frac{\frac{1+R^k_{t+1}}{1+R_t}[\Gamma'_t\left(\bar{\omega}_{t+1}\right)-\mu G'_t(\bar{\omega}_{t+1})]}{1-\frac{(1+R^k_{t+1})}{1+R_t}[\Gamma_t\left(\bar{\omega}_{t+1}\right)-\mu G_t(\bar{\omega}_{t+1})]}. \tag{A-6}$$

Recalling again the functions defined initially, we eventually obtain the form of the intermediate-good producers' FOC:

$$\frac{1-F_t\left(\bar{\omega}_{t+1}\right)}{1-\Gamma_t\left(\bar{\omega}_{t+1}\right)}=\frac{\frac{1+R^k_{t+1}}{1+Rt}[1-F_t\left(\bar{\omega}_{t+1}\right)-\mu\bar{\omega}_{t+1}F'_t(\bar{\omega}_{t+1})]}{1-\frac{1+R^k_{t+1}}{1+R_t}[\Gamma_t\left(\bar{\omega}_{t+1}\right)-\mu G_t(\bar{\omega}_{t+1})]}. \tag{A-7}$$

Finally, all firms select the same debt contract that can be represented as $(\bar{\omega}_{t+1}, L_t)$ or equivalently as (Z_{t+1}, L_t), regardless of their net worth.

Appendix B

This appendix describes the equilibrium conditions of the economy, which is characterized by 20 endogenous variables $\{C_t,\ H_t,\ \lambda_t,\ X_t,\ K_t,\ E_t,\ \kappa_t,\ P_t^E,\ R_t^k,\ r_t^x,\ Q_{K,t},\ N_t,\ R_t,\ \bar{\omega}_t,\ Y_t,\ A_t,\ W_t,\ M_t,\ I_t,\ Tr_t\}$ and two exogenous processes $\{\bar{A}_t,\ \sigma_t\}$. According to the environmental policy regime in place, an additional equation must be considered. Under a cap-and-trade regime, $E_t = \bar{E}$, while under a carbon tax, $P_t^E = \bar{P}^E$.

Households

$$\sigma_L \left(C_t^{\sigma_L}(1-H_t)^{1-\sigma_L}\right)^{-\eta} C_t^{\sigma_L-1}(1-H_t)^{1-\sigma_L} = \lambda_t \tag{B-1}$$

$$(1-\sigma_L)\left(C_t^{\sigma_L}(1-H_t)^{1-\sigma_L}\right)^{-\eta}(1-H_t)^{-\sigma_L}C_t^{\sigma_L} = W_t\lambda_t \tag{B-2}$$

$$\lambda_t = (1+R_t)\,\beta\,\mathbb{E}_t\,\lambda_{t+1} \tag{B-3}$$

Intermediate-Good Producers and Banks

$$X_t = K_t \tag{B-4}$$

$$E_t = \chi(1-\kappa_t)K_t \tag{B-5}$$

$$\theta_1\theta_2\kappa_t^{\theta_2-1} = \chi P_t^E \tag{B-6}$$

$$1+R_{t+1}^k = \frac{r_{t+1}^x + (1-\delta)Q_{K,t+1} - \theta_1(\kappa_{t+1})^{\theta_2} - P_{t+1}^E\chi(1-\kappa_{t+1})}{Q_{K,t}} \tag{B-7}$$

$$Q_{K,t}K_{t+1} = \frac{N_{t+1}}{1-\frac{1+R_{t+1}^k}{1+R_t}[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})]} \tag{B-8}$$

$$\frac{1-F_t(\bar{\omega}_{t+1})}{1-\Gamma_t(\bar{\omega}_{t+1})} = \frac{\frac{1+R_{t+1}^k}{1+R_t}[1-F_t(\bar{\omega}_{t+1}) - \mu\bar{\omega}_{t+1}F_t'(\bar{\omega}_{t+1})]}{1-\frac{1+R_{t+1}^k}{1+R_t}[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})]} \tag{B-9}$$

$$N_{t+1} = \gamma\,[1-\Gamma_{t-1}(\bar{\omega}_t)]\,(1+R_{t+1}^k)Q_{K,t-1}K_t + W_t^p \tag{B-10}$$

Final-Good Producers

$$Y_t = A_t X_t^{\alpha} {H_t}^{1-\alpha} \tag{B-11}$$

$$W_t = (1-\alpha)\frac{Y_t}{H_t} \tag{B-12}$$

$$r_t^x = \alpha\frac{Y_t}{X_t} \tag{B-13}$$

$$A_t = \bar{A}_t\,(1-D_t(M_t)) \tag{B-14}$$

Capital-Good Producers

$$K_{t+1} = (1-\delta)K_t + \left(1 - \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t \tag{B-15}$$

$$\begin{aligned} Q_{K,t} =& 1 + Q_{K,t}\frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + \gamma_I Q_{K,t}\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}} + \\ & - \gamma_I \beta\, \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} Q_{K,t+1}\left(\frac{I_{t+1}}{I_t} - 1\right)\frac{I_{t+1}^2}{I_t^2} \end{aligned} \tag{B-16}$$

where we have assumed $S(I_t/I_{t-1}) = \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$.

Pollution and Damage

$$1 - D_t(M_t) \equiv \exp\left(-\xi\left(M_t - \bar{M}\right)\right)$$

$$M_t = (1-\delta_M)\, M_{t-1} + E_t + E_t^* \tag{B-17}$$

Market Clearing

$$Y_t = I_t + C_t + \mu \int_0^{\bar{\omega}_t} \omega dF_t(\omega)(1 + R_{t+1}^k) Q_{K,t-1} K_t + \theta_1 (\kappa_t)^{\theta_2} K_t \tag{B-18}$$

$$E_t P_t^E = Tr_t \tag{B-19}$$

Exogenous Processes

$$\log \bar{A}_t = \rho_A \log \bar{A}_{t-1} + (1-\rho_A)\log A + \varepsilon_{A,t} \tag{B-20}$$

$$\log \sigma_t = \rho_\sigma \log \sigma_{t-1} + (1-\rho_\sigma)\log \sigma + \varepsilon_{\sigma,t} \tag{B-21}$$

where variables without subscript are steady-state values.

Appendix C

Starting from the model variables consumption C, investments I and net output Y^n, we define $c_t = log(C_t) - log(C)$, $i_t = log(I_t) - log(I)$ and $y_t^n = log(Y_t^n) - log(Y^n)$. We then compute the simulated moments generated by the model and compare them to those observed in the US data. Table C-1 reports the results.

Table C-1: Model and Data - Moments

	Model	Data
Standard Deviation		
σ_{y^n}	0.010	0.010
σ_i/σ_{y^n}	3.72	4.67
σ_c/σ_{y^n}	0.77	0.85
Cross-Correlations		
ρ_{i,y^n}	0.80	0.89
ρ_{c,y^n}	0.67	0.92
First-Order Autocorrelation		
ρ_{y^n}	0.79	0.90
ρ_i	0.94	0.88
ρ_c	0.67	0.86

Note: the table reports the moments generated by the model (under carbon tax) and those of the US HP-filtered quarterly data over the period 1985Q1-2019Q4, retrieved from FRED.

Appendix D

Appendix D.1

In the main text, the analysis was carried out using a measure of the welfare costs of the business cycle based on the unconditional expectation of welfare. In Table D-1, we also report the welfare costs based on the conditional expectation of welfare. As a common practice in the literature, to ensure that the economy starts from a consistent initial point in both policy regimes, we calculate the conditional expectation of welfare based on the initial state being the non-stochastic steady state. However, both of these measures for the welfare cost of the business cycle take as a reference point the deterministic steady state since they are computed as the share ς, such that $u(C(1-\varsigma), H)/(1-\beta)$ is equal to the unconditional (or conditional) expectation of welfare.

To get a taste of the welfare differences between the two environmental regimes in the long run, we can measure welfare costs using the notion of unconditional (or conditional) compensating variations. Following Lester et al. (2014), the model is solved using two sets of standard deviation values for productivity and risk shocks: high and low. The objective is to quantify the welfare costs resulting from different shock volatilities. To this end, we compute the compensating variation in consumption, which denotes the percentage by which consumption in the low-volatility situation must be reduced to achieve the same level of welfare as that in the high-volatility scenario. For simplicity, we consider the baseline calibration for the high volatility scenario, while for the low volatility scenario, we solve the model assuming that the standard deviations are half of their values in the baseline case. Also, in this case, we consider both unconditional and conditional expectation welfare to compute the welfare costs. From Table D-1 we observe that no matter what measure is used, the results show that the difference in welfare costs between the two regimes is not negligible.

Table D-1: Different Measures of Welfare Costs of the Business Cycle

	Cap-and-Trade	Carbon Tax
Unconditional Expectation	0.6178	1.5231
Conditional Expectation	0.5457	1.1294
Unconditional Compensating Variation	0.4634	1.1423
Conditional Compensating Variation	0.4093	0.8470

Appendix D.2

Table D-2 expands on Table 2 in the main text by including the case without policy. The results show that the differences from the carbon tax scenario are negligible.

Table D-2: Mean (and Volatility) for a Selection of Variables and Welfare Costs

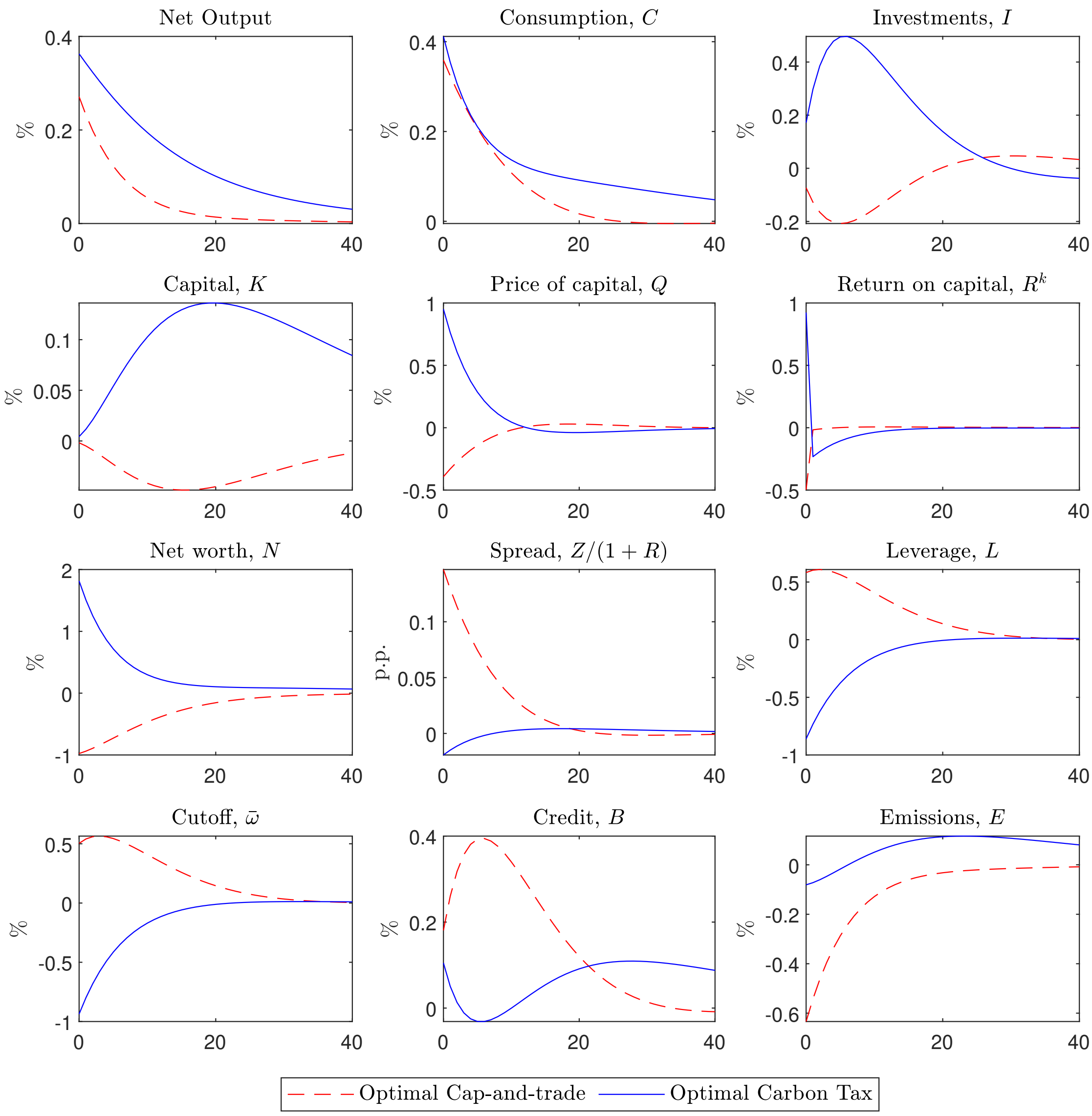
	Cap-and-Trade	Carbon Tax	No Policy
Net Output	-0.5691 (0.0189)	-2.0569 (0.0360)	-2.1219 (0.0367)
Consumption	-0.4837 (0.0115)	-1.5935 (0.0196)	-1.6339 (0.0198)
Investment	-0.9034 (0.0113)	-3.8697 (0.0236)	-3.9879 (0.0243)
Bankruptcy	0.5357 (0.0267)	0.7028 (0.0345)	0.7073 (0.0349)
Net worth	3.0069 (0.7103)	0.9986 (0.8225)	0.9229 (0.8482)
Spread	0.1573 (0.0082)	0.2728 (0.0111)	0.2735 (0.0111)
Emissions	- -	-4.1252 (0.0073)	-4.2466 (0.0074)
Carbon Price	-60.1306 (0.1781)	- -	- -
Rel. Compliance Costs	-0.0550 (0.0029)	0.0008 (0.0001)	- -
Welfare costs	0.6178	1.5231	1.5522

Note: Mean results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate, the spread and the relative compliance costs, which are reported as percentage point deviations. Standard deviations are in parentheses.

Appendix E

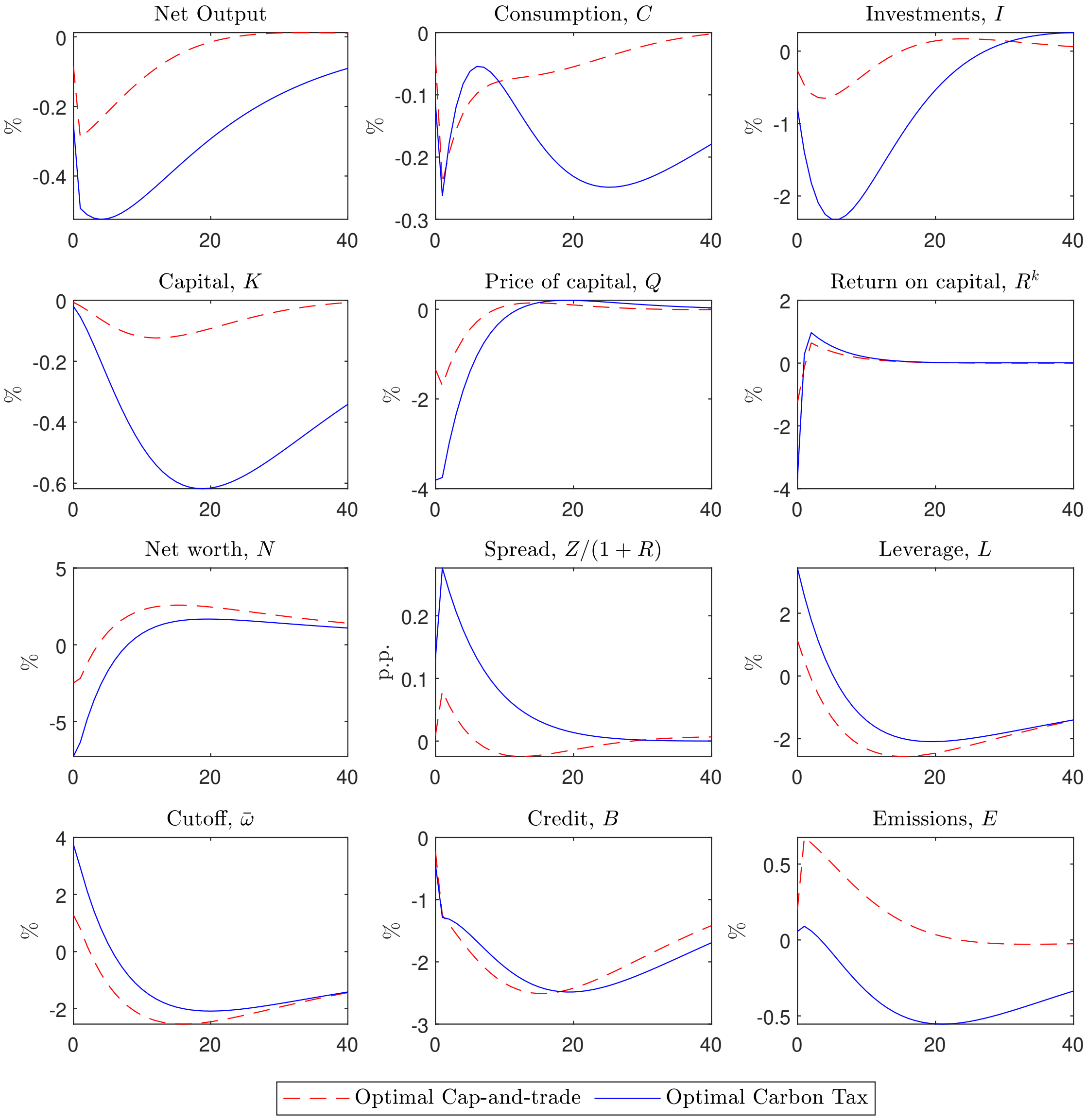
Figures E-1 and E-2 show the response of the economy to positive shocks to TFP and risk under the two optimal environmental policies (without an active macroprudential instrument).

Figure E-1: Dynamic Response to a One-Standard-Deviation TFP Shock



Note: Results are reported as percentage deviations from the initial steady state, except for the spread, which is reported as percentage point deviations from the steady state.

Figure E-2: Dynamic Response to a One-Standard-Deviation Risk Shock



Note: Results are reported as percentage deviations from the initial steady state, except for the spread, which is reported as percentage point deviations from the steady state.

Appendix F

Appendix F.1

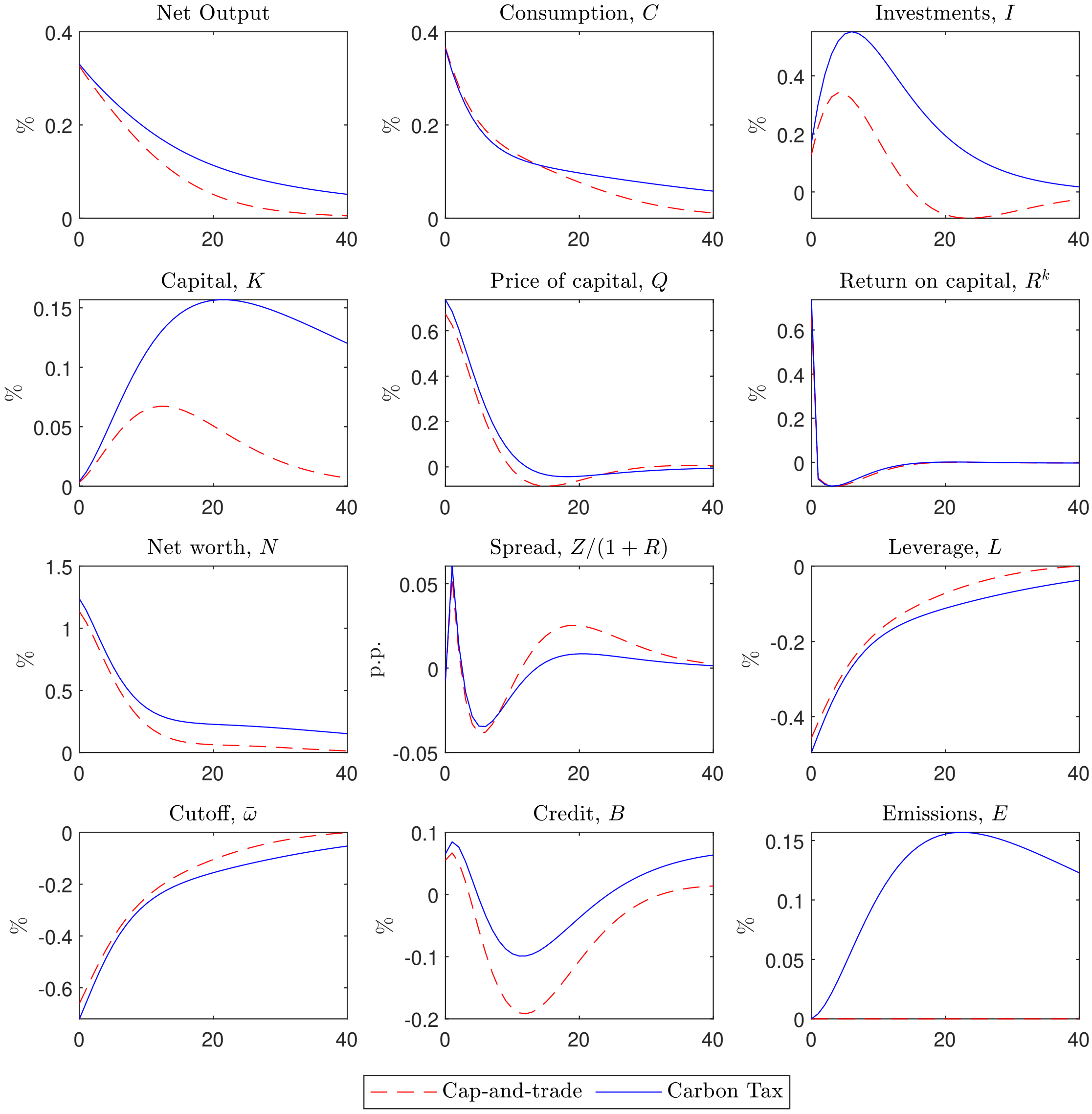
In this appendix, we report some additional results on macroprudential regulation in the form of reserve requirements. Figures F-1 and F-2 show the impulse response of the economy to positive shocks to TFP and risk under a reserve requirement that optimally responds to credit growth. Tables F-1 and F-2 report moments generated by the model under a static and optimal macroprudential policy. Finally, Tables F-3 and F-4 present two other sets of results in which the welfare costs of the business cycle are calculated under hybrid policy mixes. A first one, where the optimal macroprudential policy is conditional on a predetermined environmental rule; and a second one, where the optimal environmental policy is conditional on a predetermined macroprudential rule.

Table F-1: Mean (and Volatility) for a Selection of Variables and Welfare Costs Under Static Macroprudential Policy Rule - Reserve Requirement

	Cap-and-Trade	Carbon Tax
Net Output	-0.2007 (0.0089)	-0.4796 (0.0147)
Consumption	-0.1672 (0.0069)	-0.3864 (0.0098)
Investment	-0.3655 (0.0044)	-0.9397 (0.0089)
Bankruptcy	0.1694 (0.0083)	0.1856 (0.0088)
Net worth	5.4258 (0.3884)	5.0493 (0.3544)
Spread	0.0437 (0.0025)	0.0670 (0.0027)
Emissions	- -	-0.9951 (0.0025)
Carbon Price	-19.7031 (0.1078)	- -
Rel. Compliance Costs	-0.0182 (0.0018)	0.00014 (0.00003)
Welfare costs	0.1957	0.3863

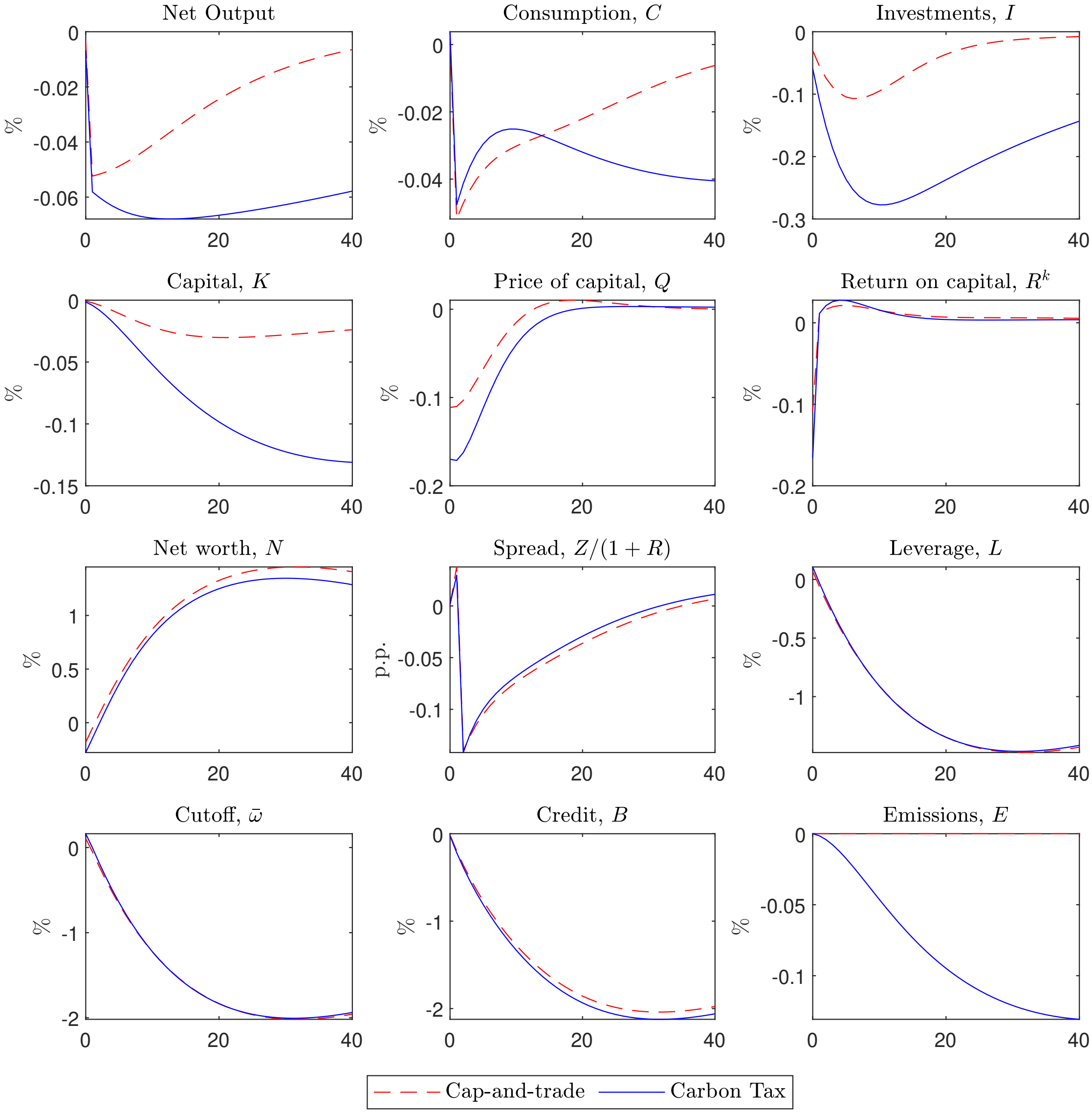
Note: Mean results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate, the spread and the relative compliance costs, which are reported as percentage point deviations. Standard deviations are in parentheses.

Figure F-1: Dynamic Response to a One-Standard-Deviation TFP Shock - Reserve Requirement



Note: The reserve requirement responds to credit growth. Results are reported as percentage deviations from the initial steady state, except for the spread, which is reported as percentage point deviations from the steady state.

Figure F-2: Dynamic Response to a One-Standard-Deviation Risk Shock - Reserve Requirement



Note: The reserve requirement responds to credit growth. Results are reported as percentage deviations from the initial steady state, except for the spread, which is reported as percentage point deviations from the steady state.

Table F-2: Mean (and Volatility) for a Selection of Variables and Welfare Costs Under Reserve Requirement Optimally Responding to Credit Growth

	Cap-and-Trade	Carbon Tax
Net Output	-0.0957 (0.0075)	-0.4256 (0.0102)
Consumption	-0.0836 (0.0063)	-0.3349 (0.0072)
Investment	-0.1550 (0.0016)	-0.8731 (0.0036)
Bankruptcy	0.1548 (0.0073)	0.1561 (0.0074)
Net worth	5.1327 (0.3847)	4.3272 (0.3547)
Spread	0.0340 (0.0047)	0.0573 (0.0045)
Emissions	- -	-0.8819 (0.0014)
Carbon Price	-26.3458 (0.0430)	- -
Rel. Compliance Costs	-0.0233 (0.0007)	0.00001 (0.00001)
Welfare costs	0.1207	0.3231

Note: Mean results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate, the spread and the relative compliance costs, which are reported as percentage point deviations. Standard deviations are in parentheses.

Table F-3: Welfare Costs of the Business Cycle under Hybrid Policy Mixes

Optimal Environmental Policy Given Reserve Requirement Policy	
	Cap-and-Trade $E_t = E\left(\frac{Y_t^n}{Y^n}\right)^{\nu}$
$\Phi_t = \Phi^{\star}\left(\frac{B_{t+1}}{B_t}\right)^{-\psi^B}$	0.1164 $\psi^B = 1.0465,\ \nu = -0.3010$
$\Phi_t = \Phi^{\star}\left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-\psi^Q}$	0.1776 $\psi^Q = 0.7220,\ \nu = -0.2345$
	Carbon Tax $P_t^E = P^E\left(\frac{Y_t^n}{Y^n}\right)^{\tau}$
$\Phi_t = \Phi^{\star}\left(\frac{B_{t+1}}{B_t}\right)^{-\psi^B}$	0.2697 $\psi^B = 0.9935,\ \tau = 44.0400$
$\Phi_t = \Phi^{\star}\left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-\psi^Q}$	0.2215 $\psi^Q = 0.6790,\ \tau = 9.4645$

Table F-4: Welfare Costs of the Business Cycle under Hybrid Policy Mixes

Optimal Reserve Requirement Policy Given Environmental Policy Rule	
	Cap-and-Trade $E_t = E\left(\frac{Y_t^n}{Y^n}\right)^{\nu}$
$\Phi_t = \Phi^{\star}\left(\frac{B_{t+1}}{B_t}\right)^{-\psi^B}$	0.1166 $\psi^B = 1.0500,\ \nu = -0.3695$
$\Phi_t = \Phi^{\star}\left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-\psi^Q}$	0.1783 $\psi^Q = 0.6699,\ \nu = -0.3695$
	Carbon Tax $P_t^E = P^E\left(\frac{Y_t^n}{Y^n}\right)^{\tau}$
$\Phi_t = \Phi^{\star}\left(\frac{B_{t+1}}{B_t}\right)^{-\psi^B}$	0.2766 $\psi^B = 1.0051,\ \tau = 24.2990$
$\Phi_t = \Phi^{\star}\left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-\psi^Q}$	0.2252 $\psi^Q = 0.7321,\ \tau = 24.2990$

Appendix F.2

In this appendix, we report some additional results on macroprudential regulation in the form of an interest rate subsidy to depositors. Table F-5 reports the steady state values for a selection of variables with and without macroprudential policy. Table F-6 shows moments of the main variables under an interest rate subsidy to depositors adjusted according to a credit growth rule.

Table F-5: Steady-State Under Macroprudential Policies

	Baseline	Reserve	Subsidy
Output Y	1.00	0.85	1.11
Investments I	0.20	0.14	0.25
Leverage L	2.01	1.73	2.12
Percent of bankrupt business p/quarter $F(\overline{\omega})$	1.50	0.32	2.11
Return spread p.p. $R^k - R$	1.51	2.49	0.99
Welfare	-62.60	-67.65	-60.08

Note: The return spread under the subsidy policy is computed as $R^k - R - s^*$.

Table F-6: Mean (and Volatility) for a Selection of Variables and Welfare Costs Under Interest Rate Subsidy Optimally Responding to Credit Growth

	Cap-and-Trade	Carbon Tax
Net Output	-0.3551 (0.0160)	-0.6018 (0.0434)
Consumption	-0.2582 (0.0110)	-0.4671 (0.0232)
Investment	-0.6810 (0.0092)	-1.0547 (0.0260)
Bankruptcy	0.1616 (0.0146)	0.1802 (0.0161)
Net worth	0.4556 (0.9182)	-0.5565 (0.8933)
Spread	0.0580 (0.0051)	0.0795 (0.0052)
Emissions	- -	-1.1168 (0.0126)
Carbon Price	-15.4899 (0.1647)	- -
Rel. Compliance Costs	-0.0153 (0.0027)	0.0002 (0.00006)
Welfare costs	0.2506	0.4706

Note: Mean results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate, the spread and the relative compliance costs, which are reported as percentage point deviations. Standard deviations are in parentheses.